In presenting the dissertation as a partial fulfillment of the requirements for an advanced degree from the Georgia Institute of Technology, I agree that the Library of the Institute shall make it available for inspection and circulation in accordance with its regulations governing materials of this type. I agree that permission to copy from, or to publish from, this dissertation may be granted by the professor under whose direction it was written, or, in his absence, by the Dean of the Graduate Division when such copying or publication is solely for scholarly purposes and does not involve potential financial gain. It is understood that any copying from, or publication of, this dissertation which involves potential financial gain will not be allowed without written permission.

3/17/65
b

TRANSVERSE VIBRATION ANALYSIS
OF A
CURVED SANDWICH PANEL

A THESIS

Presented to

The Faculty of the Graduate Division

by

Harry Eugene Plumblee, Jr.

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Engineering Mechanics

Georgia Institute of Technology

May, 1967

TRANSVERSE VIBRATION ANALYSIS
OF A
CURVED SANDWICH PANEL

Approved:

Date approved by Chairman: May 24, 1967

ACKNOWLEDGEMENTS

The author wishes to thank Dr. W. W. King for his guidance and interest which aided considerably in the completion of this analysis.

He expresses gratitude to Drs. C. E. Stoneking and G. J. Simitses for their reading of the manuscript and for their constructive criticism of the problems involved.

The author is especially grateful to Mr. K. Prince for his demonstration of speed and preciseness in typing the thesis.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

SUMMARY

The natural frequencies of free vibration of cylindrically curved honeycomb sandwich panels are investigated. The strain energy and kinetic energy are developed in terms of shell mid-surface displacements and rotation. The analysis is general from an elastic viewpoint, but simplifying assumptions applicable to lightweight aircraft structures are imposed. Two sets of mode functions, which satisfy the clamped edge boundary conditions, are used in a Rayleigh-Ritz analysis. LaGrange's equation is used in the determination of natural frequencies. Flexural modes are of primary concern; however, the analysis also yields eigenvalues for in-plane and shearing motion.

Numerical studies were conducted to show the convergence of the clamped panel eigenvalues as a function of number of terms in the mode shape approximation. Also discussed are the effects on natural frequency attributed to variations of subtended angle, modulus of elasticity, core to skin density ratio and core to skin thickness ratio.

An experimental test was conducted, but comparisons with calculations are not good because of limitations in experimentally attaining clamped edge conditions and in determination of correct values of core transverse shear moduli.

GLOSSARY OF ABBREVIATIONS AND NOTATION

A	Aspect ratio, b/ℓ
b	Panel arc length
C	Ratio of core transverse shear moduli
C_{ij}	Elastic constants (Hooke's law)
D	Plate stiffness
E	Young's modulus for isotropic material
f	Frequency
g	Ratio of core thickness to skin thickness
G_{xy}, G_{yz}	Shear moduli for orthotropic material
h_1, h_2, h_3	Honeycomb panel layer thicknesses (defined in Figure 2)
H	Ratio of core to skin mass densities
ℓ	Panel length (for simple and sandwich panel)
L	Final stiffness matrix
M_{ij}	Final mass matrix element
q_{mnk}	Generalized coordinate ($q_{mn1} = U_{mn}$, $q_{mn2} = V_{mn}$, $\ldots$ $q_{mn5} = \Phi_{mn}$)
R	Honeycomb panel mid-plane radius
S	Ratio of core shear modulus to skin Young's modulus
t	Ratio of panel length to skin thickness
T_o	Kinetic energy density
T	Kinetic energy
$\overline{u}$	Mid-plane displacement component in axial direction
$_r u_i$	Components of displacement of shell material points for r^{th} layer (defined in Equation 1)

U_o	Strain energy density
U	Strain energy
$U_{mn}, V_{mn}, W_{mn}, \Psi_{mn}, \Phi_{mn}$	Generalized coordinates for mn^{th} mode
$\bar{v}$	Mid-plane displacement component in circumferential direction
w	Mid-plane displacement component in radial, z, direction
x, y, z	Shell mid-surface curvilinear coordinate system
$X_m(x)$	Mode shape for axial direction
$Y_n(y)$	Mode shape for circumferential direction
α_m	Constant appearing in clamped mode function
β_m	Constant appearing in mode function
γ_n	Constant appearing in mode function
δ_{ij}	Kronecker delta
ϵ_i	Strain component (defined in Equation 2)
θ	Subtended angle, b/R
θ_n	Constant appearing in clamped mode function
ν	Poisson's ratio for isotropic material
ρ	Mass density
σ, σ_i	Stress
τ	Shear stress
φ	Rotation angle about x-axis
ψ	Rotation angle about y-axis
ω	Circular frequency

Ω	Non-dimensionalized circular frequency
$\lfloor \ \rfloor$	Row matrix
$\{ \ \}$	Column matrix
$[\ ]$	Rectangular matrix
$\lceil \ \rfloor$	Diagonal matrix
$[I]$	Identity matrix
$'$	Derivative with respect to argument
$\cdot$	Time derivative
$-$	Root mean square

CHAPTER I

INTRODUCTION

The purpose of this analysis is to determine the natural frequencies of a cylindrically curved sandwich panel considering both clamped and simply supported edges.

The cylindrically curved sandwich panel is a commonly used structure in aircraft design; however, no analyses exist for natural frequencies of the panel for other than simply supported edges. In a practical application, the panel edges are elastically supported. Natural frequencies for this configuration are generally bracketed by the two classical edge conditions, i.e. all edges clamped or all edges simply supported (these conditions are mathematically defined in the body of the analysis).

The complexity of the solution when elastic edges are included makes the problem impractical using available techniques. Therefore, this analysis will only include solutions for clamped and simply supported edges. If the elastic constraints are in the form of rotational or in-plane springs normal to the edges, then the analysis presented here will provide an upper and lower bound to the actual natural frequencies, within the accuracy of the Rayleigh-Ritz solution.

Although the present problem, that of determining the flexural vibration modes of a clamped curved sandwich panel, has not been solved prior to this publication, several other papers dealing with honeycomb sandwich beam and panel vibrations have been presented.

Raville, Ueng and Lei[1] present a method of determining the natural frequencies of fixed-end sandwich beams. The assumptions included homogeneity and isotropy of the thin elastic facings. Also, the core is elastic, homogeneous, orthotropic, and rigid through the thickness (i.e., $\partial w/\partial z = 0$) and continuity exists at the interfaces. A Lagrangian multiplier approach is used to satisfy the boundary conditions in the energy analysis. Comparisons with experiment show excellent agreement.

A further analysis by Ueng[2] determines the natural frequencies of flat honeycomb sandwich plates with all edges clamped. The assumptions and method of analysis are identical to those of Reference (1). The Lagrange multiplier method is also used. Experimental values vary from 5% to 10% below calculated values for clamped edges. Discrepancies are attributed to difficulties in experimentally imposing the clamped edge conditions.

Two analyses have been published which deal with curved sandwich structures. Freudenthal and Bieniek[3] determined the forced vibration characteristics of cylindrically curved sandwich plates with simply supported edges. The assumptions were similar to Ueng's.[1,2] Dissipative forces and elastic properties were accounted for in the complex modulus of elasticity. The equations of motion were written and a uniformly distributed pressure was included. An exact solution was found in terms of sine functions. No calculations or experimental confirmation were given.

Mead and Pretlove[4][5] went one step further than that presented in Reference (3) and included a variation in deflection through the thickness (i.e., $\partial w/\partial z \neq 0$). They obtained both flexural mode and "bubbling mode" frequencies. (The term "bubbling mode" was coined in Reference (4). It pertains to modes with out-of-phase motion between the inner and outer faces.) The solution was determined from the equations of motion. The flat-plate solution was readily obtained, but when curvature was imposed it was necessary to assume that the core deflection could be described accurately by the flat-plate deflection equations. The curved plate boundary conditions were imposed, however. Extensive numerical results were included.

Ballentine, Plumblee, and Schneider[6] give an analysis for curved single-layer panels and for sandwich panels with simply supported tapered edges. The single-layer panel analysis is for clamped edges. The sandwich panel theory utilizes an assumed deflection series in a conventional Rayleigh-Ritz analysis. Good agreement is shown between theory and experiment. The first measured frequency is 20% below the calculated value. All frequencies associated with higher order modes are within 15% of the calculated values.

CHAPTER II

ANALYSIS

There are many methods available for determining frequencies of free vibrations of elastic systems. Some methods lead to solutions of the differential equations of motion. The others deal with approximate solutions. One class of approximate methods is broadly categorized as "Energy Methods".

The analysis presented in this paper falls in the category of energy methods and is based on utilization of Lagrange's Equations and assumed mode shapes (Rayleigh-Ritz Method).

Assumptions

It is assumed that the materials are linearly elastic, homogeneous, and orthotropic. Exact, linear, strain-displacement relationships in cylindrical coordinates are used. It is assumed that the radial displacement does not vary through the thickness of the shell. The assumption is made that normals remain straight such that displacement due to shearing varies linearly through the thickness of the sandwich. It is also assumed that the faces of the sandwich are thin and that no transverse shearing action occurs in the facing sheets.

The configuration of the panel is shown in Figure 1. It is uniform in its plane and has three layers of material. The middle layer acts as a low density stabilizer for the outer layers, creating a lightweight panel resistant to bending.

A curvilinear coordinate system, shown in Figure 2(a), is used. The layers are: (1) for the center or core, (2) for the inner face, and (3) for the outer face. The radius of the panel is referenced to the midsurface of the core layer of the sandwich.

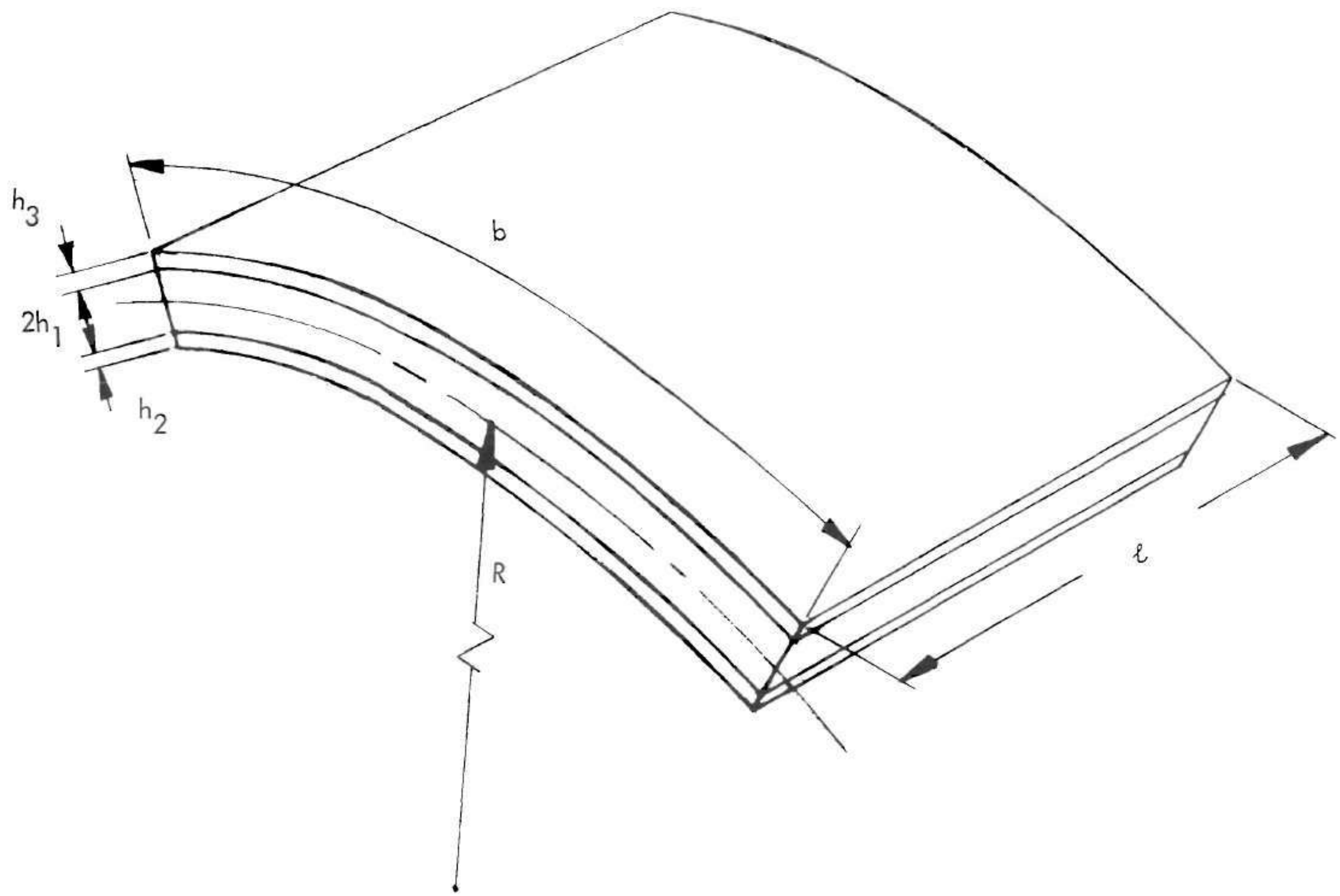

Figure 1. Curved Sandwich Panel Configuration

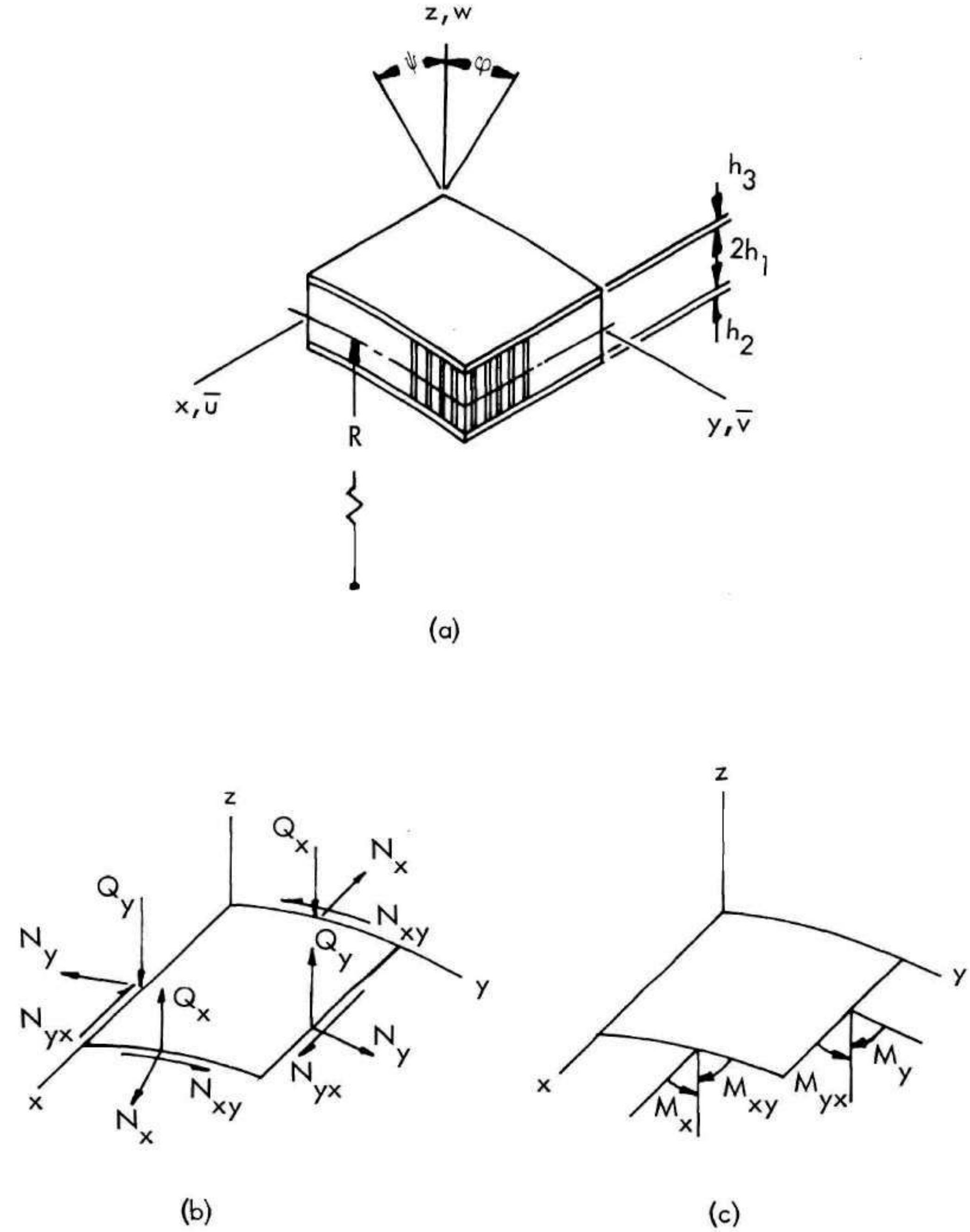

Figure 2. Sandwich Panel Coordinate System Showing the Shell Forces and Moments

Based upon these assumptions, the displacement of a given point in the panel, denoted by ${}_r u_i$, is given as

$$
\begin{array}{lll}
{}_1u_1 = \bar{u} + z\psi & {}_1u_2 = \bar{v} + z\varphi & {}_1u_3 = w \\
{}_2u_1 = \bar{u} - h_1\psi & {}_2u_2 = \bar{v} - h_1\varphi & {}_2u_3 = w \\
{}_3u_1 = \bar{u} + h_1\psi & {}_3u_2 = \bar{v} + h_1\varphi & {}_3u_3 = w
\end{array}
\tag{1}
$$

where the pre-subscript r denotes the layer, the subscript i specifies the curvilinear coordinate (1 denotes x, 2 denotes y, 3 denotes z), and $\bar{u}, \bar{v}$ are mid-surface in-plane displacements.

Energy Formulations

The first step in the analysis is the basic formulation of the strain energy and kinetic energy relationships.

Strain Energy

The strain energy density of an elastic body, without thermal stresses, is given as:

$$
U_o = \frac{1}{2} \lfloor \sigma_i \rfloor \{\varepsilon_i\} = \frac{1}{2} \lfloor \varepsilon_i \rfloor [C_{ij}] \{\varepsilon_j\}
\tag{2}
$$

where the single index ranges from 1 to 6 ($\varepsilon_1 = e_{xx}$, $\varepsilon_2 = e_{yy}$, $\varepsilon_3 = e_{zz}$, $\varepsilon_4 = 2e_{yz} = \gamma_{23} = \gamma_{xy}$, $\varepsilon_5 = 2e_{zx} = \gamma_{31} = \gamma_{zx}$, $\varepsilon_6 = 2e_{xy} = \gamma_{12} = \gamma_{xy}$). (The notation e_{xx}, e_{yy}, e_{zz}, e_{yz}, e_{zx}, e_{xy} is from Sokolnikoff[11] and ε_1, ε_2, ε_3, γ_{23}, γ_{31}, γ_{12} is used in Wang[7].) Wang[7] derives the strain-displacement relationships in generalized curvilinear coordinates in the following form:

$$\epsilon_1 = \frac{1}{A_1}\frac{\partial u_1}{\partial \xi_1} + \frac{u_2}{A_1 A_2}\frac{\partial A_1}{\partial \xi_1} + \frac{u_3}{A_1 A_3}\frac{\partial A_1}{\partial \xi_3}$$

$$\epsilon_2 = \frac{1}{A_2}\frac{\partial u_2}{\partial \xi_2} + \frac{u_1}{A_1 A_2}\frac{\partial A_2}{\partial \xi_1} + \frac{u_3}{A_2 A_3}\frac{\partial A_2}{\partial \xi_3}$$

$$\epsilon_3 = \frac{1}{A_3}\frac{\partial u_3}{\partial \xi_3} + \frac{u_2}{A_2 A_3}\frac{\partial A_3}{\partial \xi_2} + \frac{u_1}{A_1 A_3}\frac{\partial A_3}{\partial \xi_1} \tag{3}$$

$$\epsilon_4 = \gamma_{23} = \gamma_{32} = \frac{A_3 \partial}{A_2 \partial \xi_2}\left(\frac{u_3}{A_3}\right) + \frac{A_2}{A_3}\frac{\partial}{\partial \xi_3}\left(\frac{u_2}{A_2}\right)$$

$$\epsilon_5 = \gamma_{13} = \gamma_{31} = \frac{A_1}{A_3}\frac{\partial}{\partial \xi_3}\left(\frac{u_1}{A_1}\right) + \frac{A_3}{A_1}\frac{\partial}{\partial \xi_1}\left(\frac{u_3}{A_3}\right)$$

$$\epsilon_6 = \gamma_{12} = \gamma_{21} = \frac{A_2}{A_1}\frac{\partial}{\partial \xi_1}\left(\frac{u_2}{A_2}\right) + \frac{A_1}{A_2}\frac{\partial}{\partial \xi_2}\left(\frac{u_1}{A_1}\right)$$

For the cylindrical-curvilinear coordinate system

$$A_1 = A_3 = 1, \quad A_2 = R(1 + z/R), \quad \xi_1 = x, \quad \xi_2 = y/R, \quad \xi_3 = z \; . \tag{4}$$

Substitution of Equations (4) and (1) into (3) yields the general expressions for the strain components as shown in Equation (5) below:

$$\epsilon_1 = \frac{\partial \bar{u}}{\partial x} + z\,\frac{\partial \psi}{\partial x}$$

$$\epsilon_2 = \frac{w}{R+z} + \frac{R}{R+z}\left(\frac{\partial \bar{v}}{\partial y} + z\,\frac{\partial \varphi}{\partial y}\right) \tag{5}$$

$$\epsilon_3 = \frac{\partial w}{\partial z} = 0 \text{ (one of the basic assumptions)}$$

$$\epsilon_4 = \frac{-\bar{v}}{R+z} + \frac{R}{R+z}\left(\frac{\partial w}{\partial y} + \varphi\right)$$

$$\epsilon_5 = \frac{\partial w}{\partial x} + \psi \qquad (5)\ \text{(cont'd)}$$

$$\epsilon_6 = \frac{\partial \bar{v}}{\partial x} + z\frac{\partial \varphi}{\partial x} + \frac{R}{R+z}\left(\frac{\partial \bar{u}}{\partial y} + z\frac{\partial \psi}{\partial y}\right)$$

The strain-displacement relationships for each layer of the sandwich panel are written in matrix notation for convenience in presentation and manipulation. In this form the strain-displacement relationships are:

a) For the core

$$\{{}_1\epsilon_i\} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & \frac{z}{h_1}\frac{\partial}{\partial x} & 0 \\ 0 & \frac{R}{R+z}\frac{\partial}{\partial y} & \frac{1}{R+z} & 0 & \frac{zR}{h_1(R+z)}\frac{\partial}{\partial y} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{R+z} & \frac{R}{R+z}\frac{\partial}{\partial y} & 0 & \frac{R}{h_1(R+z)} \\ 0 & 0 & \frac{\partial}{\partial x} & \frac{1}{h_1} & 0 \\ \frac{R}{R+z}\frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 & \frac{zR}{h_1(R+z)}\frac{\partial}{\partial y} & \frac{z}{h_1}\frac{\partial}{\partial x} \end{bmatrix} \begin{Bmatrix} \bar{u} \\ \bar{v} \\ w \\ h_1\psi \\ h_1\varphi \end{Bmatrix} \qquad (6)$$

b) For the inner face sheet ($z = -h_1$)

$$\{{}_2\epsilon_i\} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & -\frac{\partial}{\partial x} & 0 \\ 0 & \frac{R}{R+z}\frac{\partial}{\partial y} & \frac{1}{R+z} & 0 & \frac{-R}{R+z}\frac{\partial}{\partial y} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{R+z} & \frac{R}{R+z}\frac{\partial}{\partial y} & 0 & \frac{1}{R+z} \\ 0 & 0 & \frac{\partial}{\partial x} & 0 & 0 \\ \frac{R}{R+z}\frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 & \frac{-1}{R+z}\frac{\partial}{\partial y} & -\frac{\partial}{\partial x} \end{bmatrix} \begin{Bmatrix} \bar{u} \\ \bar{v} \\ w \\ h_1\psi \\ h_1\varphi \end{Bmatrix} \tag{7}$$

c) and for the outer face sheet ($z = h_1$)

$$\{{}_3\epsilon_i\} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & \frac{\partial}{\partial x} & 0 \\ 0 & \frac{R}{R+z}\frac{\partial}{\partial y} & \frac{1}{R+z} & 0 & \frac{R}{R+z}\frac{\partial}{\partial y} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{R+z} & \frac{R}{R+z}\frac{\partial}{\partial y} & 0 & -\frac{1}{R+z} \\ 0 & 0 & \frac{\partial}{\partial x} & 0 & 0 \\ \frac{R}{R+z}\frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 & \frac{R}{R+z}\frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \begin{Bmatrix} \bar{u} \\ \bar{v} \\ w \\ h_1\psi \\ h_1\varphi \end{Bmatrix} \tag{8}$$

It will be assumed throughout the analysis that the core layer is a thick cylindrical panel with general orthotropic elastic properties. Chu(12) showed that, by using the strain-displacement relationships of Equation (5) to develop equations of motion, the frequencies of free vibration are accurate for shell thicknesses equal to one-fourth the radius of curvature, if the wavelength of the natural mode is considerably greater than the shell thickness.

The elastic coefficient matrix for an orthotropic elastic medium[11] is:

$$[{}_1C_{ij}] = \begin{bmatrix} {}_1C_{11} & {}_1C_{12} & {}_1C_{13} & 0 & 0 & 0 \\ {}_1C_{21} & {}_1C_{22} & {}_1C_{23} & 0 & 0 & 0 \\ {}_1C_{31} & {}_1C_{32} & {}_1C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & {}_1C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & {}_1C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & {}_1C_{66} \end{bmatrix} \tag{9}$$

The elastic coefficient matrices for the thin faces are

$$[{}_2C_{ij}] = \begin{bmatrix} {}_2C_{11} & {}_2C_{12} & 0 & 0 & 0 & 0 \\ {}_2C_{21} & {}_2C_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & {}_2C_{66} \end{bmatrix} \tag{10}$$

$$[{}_3C_{ij}] = \begin{bmatrix} {}_3C_{11} & {}_3C_{12} & 0 & 0 & 0 & 0 \\ {}_3C_{21} & {}_3C_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & {}_3C_{66} \end{bmatrix} \tag{10 (cont'd)}$$

The strain energy of the panel is determined by integrating the strain energy density of each panel layer over its respective volume and summing the result. The method for performing this manipulation is only indicated at this point in the analysis. The strain energy of the panel is

$$U = \frac{1}{2}\iint_S \Big\{ \int_{-h_1}^{h_1} \lfloor {}_1\varepsilon_i \rfloor [C_{ij}] \{{}_1\varepsilon_j\}\, dz \tag{11}$$

$$+ \int_{-h_2-h_1}^{-h_1} \lfloor {}_2\varepsilon_i \rfloor [{}_2C_{ij}] \{{}_2\varepsilon_j\}\, dz + \int_{h_1}^{h_1+h_3} \lfloor {}_3\varepsilon_i \rfloor [{}_3C_{ij}] \{{}_3\varepsilon_j\}\, dz \Big\}\, dS$$

Kinetic Energy

The kinetic energy density of a given layer in the composite structure is:

$${}_rT_o = \frac{{}_r\rho}{2} \lfloor {}_r\dot{u}_i \rfloor \{{}_r\dot{u}_j\} \tag{12}$$

Substitution of the displacement relationships of Equation (1) and subsequent integration over the volume yields the kinetic energy in terms of reference surface displacements and rotations:

$$T = \frac{1}{2}\iint\limits_S \Big\{ \int_{-h_1}^{h_1} {}_1\rho \lfloor {}_1\dot{u}_i \rfloor \{ {}_1\dot{u}_i \} \, dz \; \int_{-h_1-h_2}^{-h_1} {}_2\rho \lfloor {}_2\dot{u}_i \rfloor \{ {}_2\dot{u}_i \} \, dz \qquad (13)$$
$$+ \int_{h_1}^{h_1+h_3} {}_3\rho \lfloor {}_3\dot{u}_i \rfloor \{ {}_3\dot{u}_i \} \, dz \Big\} \, dS$$

Boundary Conditions

The procedure for determining boundary conditions consists of a first-order variation of the strain and kinetic energy.

Chu(12) derived the equations of motion for a three-layer circular cylindrical sandwich shell using this method. The strain and kinetic energies were derived in terms of the shell stress components and displacement components. Hamilton's principle was applied and the differential equations and boundary conditions were determined in terms of the stress components and the displacements.

The boundary conditions for a longitudinal boundary (y = constant) required that one member of each of the following products be specified:

$$N_{yx}\,\bar{u},\; N_y\bar{v},\; Q_y\, w,\; \left[{}_1M_{yx} + h_1({}_3N_{yx} - {}_2N_{yx})\right]\psi$$
$$\text{and}\left[{}_1M_y + h_1({}_3N_y - {}_2N_y)\right]\varphi \, .$$

For a circumferential boundary (x = constant), the products were:

$$N_x \bar{u},\ N_{xy} \bar{v},\ Q_x w,\ [{}_1M_x + h_1({}_3N_x - {}_2N_x)]\ \psi\ .$$

$$\text{and}\ [{}_1M_{xy} + h_1({}_3N_{xy} - {}_2N_{xy})]\ \varphi\ .$$

The stresses and moments required in these boundary conditions are shown in Figures 2(b) and 2(c).

For this analysis, the classic boundary conditions for simply supported edges are given as:

a) $x = 0$ and ℓ

natural boundary conditions

$$N_{yx} = 0,\ [{}_1M_{yx} + h_1({}_3N_{yx} - {}_2N_{yx})] = 0$$

forced (or geometric) boundary conditions

$$\bar{v} = w = \varphi = 0$$

b) $y = 0$ and b

natural boundary conditions

$$N_{xy} = 0,\ [{}_1M_{xy} + h_1({}_3N_{xy} - {}_2N_{xy})] = 0$$

forced (or geometric) boundary conditions

$$\bar{u} = w = \psi = 0$$

The boundary conditions for a panel with clamped edges are:

$$\bar{u} = \bar{v} = w = \psi = \varphi = 0$$

on all edges.

An exact solution can be obtained for simply supported edges. The displacement functions for the exact solution are given as

$$\bar{u} = \sum_n \sum_m \frac{1}{\beta_m} U_{mn}(t)\; X'_m(x)\; Y_n(y)$$

$$\bar{v} = \sum_n \sum_m \frac{1}{\gamma_n} V_{mn}(t)\; X_m(x)\; Y'_n(y)$$

$$w = \sum_n \sum_m W_{mn}(t)\; X_m(x)\; Y_n(y) \tag{14}$$

$$\psi = \sum_n \sum_m \frac{1}{\beta_m} \Psi_{mn}(t)\; X'_m(x)\; Y_n(y)$$

$$\varphi = \sum_n \sum_m \frac{1}{\gamma_n} \Phi_{mn}(t)\; X_m(x)\; Y'_n(y)$$

The mode shape functions $X_m(x)$ and $Y_n(y)$ are

$$X_m(x) = \sin \beta_m x \qquad Y_n(y) = \sin \gamma_n y$$

$$\beta_m = \frac{m\pi}{\ell} \qquad \gamma_n = \frac{n\pi}{b}$$

Each set of terms in Equation (14) satisfies the differential equations of motion.

Approximate Free Vibration Analysis for Clamped Boundaries

The Rayleigh-Ritz method is employed in the development of an approximate free vibration analysis for clamped boundaries. The Rayleigh-Ritz method requires that the deflection of the elastic system be approximated by a sequence of functions which converges in the mean-square sense to the exact deflection. The deflection functions are required to satisfy the forced boundary conditions and the sequence is also required to form a complete set.

If the assumed mode functions satisfy not only the geometric boundary conditions, but also fulfill the requirements of all the natural boundary conditions, convergence is much more rapid. In the case of simply supported boundaries, the exact solutions can be used.

It is known that convergence is obtained with fewer terms if the assumed mode closely approximates the deflection shape, provided completeness conditions are satisfied. One set of mode functions used for the clamped edge analysis utilizes the clamped beam functions.

The mode functions are

$$
\begin{aligned}
X_m(x) &= \text{Cosh}\,\beta_m x - \text{Cos}\,\beta_m x - \alpha_m(\text{Sinh}\,\beta_m x - \text{Sin}\,\beta_m x) \\
Y_n(y) &= \text{Cosh}\,\gamma_n y - \text{Cos}\,\gamma_n y - \theta_n(\text{Sinh}\,\gamma_n y - \text{Sin}\,\gamma_n y) \\
\alpha_m &= \frac{\text{Cosh}\,\beta_m \ell - \text{Cos}\,\beta_m \ell}{\text{Sinh}\,\beta_m \ell - \text{Sin}\,\beta_m \ell} \\
\theta_n &= \frac{\text{Cosh}\,\gamma_n b - \text{Cos}\,\gamma_n b}{\text{Sinh}\,\gamma_n b - \text{Sin}\,\gamma_n b}
\end{aligned}
\tag{15}
$$

and β_m and γ_n are determined from the following relationships

$$\text{Cosh}\,\beta_m \ell\ \text{Cos}\,\beta_m \ell = 1$$

$$\text{Cosh}\,\gamma_n b\ \text{Cos}\,\gamma_n b = 1$$

Use of the functions of Equation (15) is justified because they can be used directly in the displacement relationships of Equation (14). Therefore, use of the beam functions permits simultaneous solution for two sets of boundary conditions using general mode shape notation. Substitution of the actual mode shapes is never accomplished, since integrals of the beam functions have been tabulated by Felgar.[13]

Since the beam functions are eigenfunctions, they satisfy the requirements for a complete set as shown by Courant and Hilbert.[14] The completeness of the first derivative of the beam function has not been demonstrated, but the first derivative of the clamped beam function has been used by many research investigators in panel and shell vibration analyses (Scruggs,[9] Ballentine,[6] and Sewall[15]).

A parallel analysis was conducted using sine functions to satisfy the clamped boundary conditions. The set of deflection functions used are

$$\begin{aligned}
\bar{u} &= \sum_m \sum_n U_{mn} X_m(x) Y_n(y) \\
\bar{v} &= \sum_m \sum_n V_{mn} X_m(x) Y_n(y) \\
w &= \sum_m \sum_n W_{mn} X_m(x) Y_n(y) \\
\psi &= \sum_m \sum_n \Psi_{mn} X_m(x) Y_n(y) \\
\varphi &= \sum_m \sum_n \Phi_{mn} X_m(x) Y_n(y)
\end{aligned} \tag{16}$$

The mode functions are

$$X_m(x) = \text{Sin}\, \beta_m x \qquad Y_m(y) = \text{Sin}\, \gamma_n y$$

$$\beta_m = \frac{m\pi}{\ell} \qquad \gamma_n = \frac{n\pi}{b}$$

The manipulations required to calculate natural frequencies are shown only for the exact solution of simply supported boundaries and the beam function - clamped boundary solution, since in generalized form (Equation (14)) the displacement equations are identical. The final equations for the clamped boundary using sine function approximations are presented in Appendix I.

Strain Energy

It would be virtually impossible to determine the strain energy without some form of bookkeeping system, because of the number of terms and manipulations involved in accomplishing the requirements of Equations (11) and (13). Therefore, to provide some degree of orderliness, matrix notation is utilized throughout the remainder of the analysis.

The equations for strain (Equations (6), (7), and (8)) are written as:

$$\{{}_r\epsilon_i\} = [{}_rA_{ik}] \, \{u_k\} \tag{17}$$

where u_k represents the displacements (i.e., $u_1 = \overline{u}$, $u_2 = \overline{v}$, $u_3 = w$, $u_4 = h_1\psi$, $u_5 = h_1\varphi$). $[A_{ik}]$ is an operator matrix and the pre-subscript r denotes the layer.

The assumed displacements of Equation (14) may also be represented in matrix form and are:

$$\{u_k\} = \left[\lfloor B_{mn} \rfloor_{kk} \right] \left\{ \{q_{mn}\}_k \right\} \tag{18}$$

$$[B] = \begin{bmatrix} \left\lfloor \frac{X'_m}{\beta_m} Y_n \right\rfloor & 0 & 0 & 0 & 0 \\ 0 & \left\lfloor X_m \frac{Y'_n}{Y_n} \right\rfloor & 0 & 0 & 0 \\ 0 & 0 & \lfloor X_m Y_n \rfloor & 0 & 0 \\ 0 & 0 & 0 & \left\lfloor \frac{X'_m}{\beta_m} Y_n \right\rfloor & 0 \\ 0 & 0 & 0 & 0 & \left\lfloor X_m \frac{Y'_n}{Y_n} \right\rfloor \end{bmatrix} \tag{18 (cont'd)}$$

Substitution of Equation (18) into Equation (17) yields:

$$\{{}_r\epsilon_i\} = [{}_rA_{jk}] \left[\lfloor B_{mn} \rfloor_{kk} \right] \left\{ \{q_{mn}\}_k \right\} \tag{19}$$

or performing the operations required by the operator matrix $[A]$, the strain is:

$$\{{}_r\epsilon_i\} = \left[{}_r\lfloor D_{mn} \rfloor_{jk} \right] \left\{ \{q_{mn}\}_k \right\} \tag{20}$$

Now, substituting Equation (20) into Equation (2) gives strain energy in terms of the generalized coordinates:

$${}_rU_o = \frac{1}{2} \left\lfloor \lfloor q_{st} \rfloor_\ell \right\rfloor \left[{}_r\{D_{st}\}_{\ell i} \right] \left[{}_rC_{ij} \right] \left[{}_r\lfloor D_{mn} \rfloor_{jk} \right] \left\{ \{q_{mn}\}_k \right\} \tag{21}$$

After performing the indicated multiplications, the strain energy density is

$$ {}_r U_o = \frac{1}{2} \left\lfloor \lfloor q_{st} \rfloor_\ell \right\rfloor \left[{}_r\left[E_{stmn} \right]_{\ell k} \right] \left\{ \{ q_{mn} \}_k \right\} \tag{22} $$

Since the objective is to obtain total strain energy of the panel, it is necessary to integrate the strain energy density over the volume. The most convenient manipulation is that of integrating the strain energy density through the thickness for each layer and then summing over the layers. After this operation, the strain energy surface density is integrated over the panel surface area. The strain energy surface density is given as

$$ S = \int_{-h_1}^{h_1} {}_1U_o \, dz + \int_{-h_1-h_2}^{-h_1} {}_2U_o \, dz + \int_{h_1}^{h_1+h_3} {}_3U_o \, dz = {}_1S + {}_2S + {}_3S \tag{23} $$

The ${}_rS$ matrix is mathematically represented as follows:

$$ {}_rS = \left\lfloor \lfloor q_{rs} \rfloor_\ell \right\rfloor \left[{}_r\left[F_{stmn} \right]_{\ell k} \right] \left\{ \{ q_{mn} \}_k \right\} \tag{24} $$

After computation, the F's are summed over r, resulting in

$$ \left[G_{stmn} \right]_{\ell k} = \sum_{r=1}^{3} {}_r\left[F_{stmn} \right]_{\ell k} \tag{25} $$

Approximations used in integration are listed on the following page.

$$\int_{-h_1}^{h_1} \frac{dz}{R+z} = \ln\left(\frac{R+h_1}{R-h_1}\right) \approx \frac{2h_1}{R}\left(1+\frac{h_1^2}{3R^2}\right) = \frac{2h_1}{R} H_1$$

$$\int_{-h_1}^{h_1} \frac{z\,dz}{R+z} = 2h_1 - R\ln\left(\frac{R+h_1}{R-h_1}\right) \approx -\frac{2}{3}\frac{h_1^3}{R^2}$$

$$\int_{-h_1}^{h_1} \frac{z^2 dz}{R+z} = -2Rh_1 - R^2\ln\left(\frac{R+h_1}{R-h_1}\right) \approx \frac{2}{3}\frac{h_1^3}{R}$$

$$\int_{-h_1}^{h_1} \frac{dz}{(R+z)^2} = -\frac{1}{R+h_1} - \frac{1}{R-h_1} \approx \frac{2h_1}{R^2}\left(1-\frac{h_1^2}{R^2}\right) = \frac{2h_1}{R^2} H_2$$

$$\int_{-h_1}^{h_1} \frac{z\,dz}{(R+z)^2} = \frac{R}{R+h_1} - \frac{R}{R-h_1} - \ln\left(\frac{R+h_1}{R-h_1}\right) \approx -\frac{4}{3}\frac{h_1^3}{R^3} \tag{26}$$

$$\int_{-h_1}^{h_1} \frac{z^2 dz}{(R+z)^2} = 2h_1 - \frac{R^2}{R+h_1} + \frac{R^2}{R-h_1} - 2R\ln\left(\frac{R+h_1}{R-h_1}\right) \approx \frac{2}{3}\frac{h_1^3}{R^2}$$

$$\int_{-h_1-h_2}^{-h_1} \frac{dz}{R+z} \approx \frac{h_2}{R}\left(1 - \frac{2h_1+h_2}{2R} + \frac{3h_1^2+3h_1h_2+h_2^2}{3R^2}\right) = \frac{h_2}{R} H_3$$

$$\int_{-h_1-h_2}^{-h_1} \frac{dz}{(R+z)^2} \approx \frac{h_2}{R^2}\left(1 - \frac{2h_1+h_2}{R} + \frac{3h_1^2+3h_1h_2+h_2^2}{R^2}\right) = \frac{h_2}{R^2} H_4$$

$$\int_{h_1}^{h_1+h_3} \frac{dz}{R+z} \approx \frac{h_3}{R}\left(1 - \frac{2h_1+h_3}{2R} + \frac{3h_1^2+3h_1h_3+h_3^2}{3R^2}\right) = \frac{h_3}{R} H_5$$

$$\int_{h_1}^{h_1+h_3} \frac{dz}{R+z} \approx \frac{h_3}{R^2}\left(1 - \frac{2h_1+h_3}{R} + \frac{3h_1^2+3h_1h_3+h_3^2}{R^2}\right) = \frac{h_3}{R^2} H_6 \quad \text{(26) (cont'd)}$$

The final step in the derivation of strain energy is the integration of strain energy surface density over the panel area. This integration is indicated in Equation (11). The integrals for the simply supported mode functions are given as:

$$\int_0^{\ell} X_s\, X_m\, dx = \delta_{sm} \frac{\ell}{2} \qquad = \ell\ {}_1M_{sm}$$

$$\int_0^{\ell} X_s'\, X_m'\, dx = \delta_{sm}\, \beta_m^2 \frac{\ell}{2} \qquad = \frac{{}_2M_{sm}}{\ell}$$

$$\int_0^{\ell} X_s''\, X_m''\, dx = \delta_{sm}\, \beta_m^4 \frac{\ell}{2} \qquad = \beta_m^4\, \ell\ {}_1M_{sm}$$

$$\int_0^{\ell} X_s''\, X_m\, dx = \int_0^{\ell} X_s\, X_m''\, dx = -\frac{{}_2M_{sm}}{\ell}$$

$$\text{(27)}$$

$$\int_0^{b} Y_t\, Y_n\, dy = \delta_{tn} \frac{b}{2} \qquad = b\ {}_1N_{tn}$$

$$\int_0^{b} Y_t'\, Y_n'\, dy = \delta_{tn} \frac{\gamma_n^2\, b}{2} \qquad = \frac{{}_2N_{tn}}{b}$$

$$\int_0^{b} Y_t''\, Y_n''\, dy = \delta_{tn} \frac{\gamma_n^4\, b}{2} \qquad = \gamma_n^4\, b\ {}_1N_{tn}$$

$$\int_0^{b} Y_t''\, Y_n\, dy = \int_0^{b} Y_t\, Y_n''\, dy = -\frac{{}_2N_{tn}}{b}$$

It is noted that since these functions are exact solutions, they are orthogonal functions. The generalized nomenclature for the integrals (${}_iM_{sm}$, ${}_iN_{tn}$) is used for both edge conditions. The value of the M's and N's will be determined by the edge condition.

$$\int_0^{\ell} X_s X_m \, dx = \delta_{sm} \ell \qquad = \ell \; {}_1M_{sm}$$

$$\int_0^{\ell} X'_s X'_m \, dx = \delta_{sm} \alpha_m \beta_m (\alpha_m \beta_m \ell - 2) + (1 - \delta_{sm}) \frac{4\beta_s^2 \beta_m^2 (\alpha_m \beta_m - \alpha_s \beta_s)}{\beta_m^4 - \beta_s^4} \left[1 + (-1)^{m+s}\right] = \frac{{}_2M_{sm}}{\ell}$$

$$\int_0^{\ell} X''_s X''_m \, dx = \delta_{sm} \beta_m^4 \ell \qquad = \beta_m^4 \ell \; {}_1M_{sm}$$

$$\int_0^{\ell} X''_s X_m \, dx = \int_0^{\ell} X_s X''_m \, dx \qquad = - \frac{{}_2M_{sm}}{\ell} \tag{28}$$

$$\int_0^{b} Y_t Y_n \, dy = \delta_{tn} b \qquad = b \; {}_1N_{tn}$$

$$\int_0^{b} Y'_t Y'_n \, dy = \delta_{tn} \theta_n \gamma_n (\theta_n \gamma_n b - 2) + (1 - \delta_{tn}) \frac{4\gamma_t^2 \gamma_n^2 (\theta_t \gamma_t - \theta_n \gamma_n)}{\gamma_n^4 - \gamma_t^4} \left[1 + (-1)^{t+n}\right] = \frac{{}_2N_{tn}}{b}$$

$$\int_0^{b} Y''_t Y''_n \, dy = \delta_{tn} \gamma_n^4 b \qquad = \gamma_n^4 b \; {}_1N_{tn}$$

$$\int_0^b Y_t \, Y_n'' \, dy = \int_0^b Y_t'' \, Y_n \, dy \qquad = -\frac{{}_2N_{tn}}{b} \qquad (28)$$

(cont'd)

The integrals required for clamped edges, using the sine functions appear in the following equation

$$\int_0^{\ell} X_s \, X_m \, dx = \delta_{sm} \frac{\ell}{2} \qquad = \ell \; {}_1M_{sm}$$

$$\int_0^{\ell} X_s' \, X_m \, dx = -\int_0^{\ell} X_s \, X_m' \, dx$$

$$= (1 - \delta_{sm}) \frac{ms}{s^2 - m^2} \left[1 - (-1)^{m+s} \right] = {}_2M_{sm}$$

$$\int_0^{\ell} X_s \, X_m \, dx = \delta_{sm} \, \beta_m^2 \frac{\ell}{2} \qquad = \beta_m^2 \, \ell \; {}_1M_{sm}$$

(29)

$$\int_0^b Y_t \, Y_n \, dy = \delta_{tn} \frac{b}{2} \qquad = b \; {}_1N_{tn}$$

$$\int_0^b Y_t' \, Y_n \, dy = -\int_0^b Y_t \, Y_n' \, dy$$

$$= (1 - \delta_{tn}) \frac{tn}{t^2 - n^2} \left[1 - (-1)^{t+n} \right] = {}_2N_{tn}$$

$$\int_0^b Y_t' \, Y_n' \, dy = \delta_{tn} \, \gamma_n^2 \frac{b}{2} \qquad = \gamma_n^2 \, b \; {}_1N_{tn}$$

After evaluation of the surface integrals, the strain energy is:

$$U = \lfloor \lfloor q_{rs} \rfloor_{\ell} \rfloor \left[\left[K_{stmn} \right]_{\ell k} \right] \left\{ \{ q_{mn} \}_{k} \right\} \tag{30}$$

This completes the derivation of strain energy, but from the standpoint of evaluation for realistic aircraft construction, some simplifying assumptions may be made. These include:

(1) For cores made of lightweight honeycomb, the elastic constants are approximated by:

$${}_1C_{11} = {}_1C_{12} = {}_1C_{21} = {}_1C_{22} = {}_1C_{66} = 0$$

$${}_1C_{44} = G_{yz}$$

$${}_1C_{55} = G_{xz}$$

(2) If the facing sheets are of sheet metal, with both sides of the same material, the elastic properties are assumed to be isotropic and the elastic constants are:

$${}_2C_{11} = {}_3C_{11} = \frac{E}{1 - \nu^2}$$

$${}_2C_{12} = {}_3C_{12} = {}_2C_{21} = {}_3C_{21} = \frac{\nu E}{1 - \nu^2}$$

$${}_2C_{22} = {}_3C_{22} = \frac{E}{1 - \nu^2}$$

$${}_2C_{66} = {}_3C_{66} = \left(\frac{1 - \nu}{2}\right) \frac{E}{1 - \nu^2}$$

(3) For convenience in manipulation and presentation, it is assumed that $h_2 = h_3$.

(4) It is assumed that the core thickness is very much less than the radius of curvature. In aircraft structural applications, h_1/R will usually be less than 0.01. It is also tacitly assumed in the derivation, in the statement of the strains, that $h_2 << h_1$.

These simplifying assumptions are introduced into the final equations, and the general terms of Equation (30) are not shown for sake of brevity.

Another simplification in presentation of the theory can be obtained by non-dimensionalization. The parameters chosen for this operation are

$\theta = b/R$	(Subtended Angle)
$t = \ell/h_2$	(Ratio of Panel Length to Skin Thickness)
$A = b/\ell$	(Aspect Ratio)
$g = h_1/h_2$	(Ratio of Core Thickness to Skin Thickness)
$S = \dfrac{(1 - \nu^2)G_{yz}}{E}$	(Ratio of Core Shear Modulus to Skin Bending Modulus)
$C = G_{xz}/G_{yz}$	(Ratio of Core Transverse Shear Moduli for Orthotropic Material)

Introduction of these parameters reduces the number of independent variables from 8 to 6; a significant reduction for evaluation purposes. A natural parameter to include in the non-dimensionalization of frequency is the stretching stiffness of the faces, $Eh_2/(1 - \nu^2)$. Upon factoring out this parameter and introducing the above non-dimensional variables and simplifying assumptions, the final version of the strain energy matrix is derived and is denoted as the L matrix. The terms in the L matrix appear below. Terms below the diagonal are not given, since the matrix is symmetric. Again, this stiffness matrix is for simply supported and clamped boundaries (assuming beam functions). The stiffness matrix for sine function - clamped edges is given in the Appendix.

$$[L_{stmn}]_{11} = A\left[(\beta_m \ell)^2 \, {}_{1}M_{sm}\left[{}_{1}N_{tn}\right]\right] + \left(\frac{1-\nu}{2}\right)\frac{1}{A}\left[\frac{{}_{2}M_{sm}}{(\beta_m \ell)(\beta_s \ell)}\left[{}_{2}N_{tn}\right]\right]$$

$$[L_{stmn}]_{12} = \left(\frac{1+\nu}{2}\right)\left[\frac{{}_{2}M_{sm}}{\beta_s \ell}\left[\frac{{}_{2}N_{tn}}{\gamma_n b}\right]\right]$$

$$[L_{stmn}]_{13} = -\nu\theta\left[\frac{{}_{2}M_{sm}}{\beta_s \ell}\left[{}_{1}N_{tn}\right]\right]$$

$$[L_{stmn}]_{14} = 0$$

$$[L_{stmn}]_{15} = 0$$

$$[L_{stmn}]_{22} = \frac{1}{A}\left[{}_{1}M_{sm}\left[(\gamma_n b)^2 \, {}_{1}N_{tn}\right]\right] + Sg\,\frac{\theta^2}{A}\left[{}_{1}M_{sm}\left[\frac{{}_{2}N_{tn}}{(\gamma_t b)(\gamma_n b)}\right]\right] \qquad (31)$$

$$+ \left(\frac{1-\nu}{2}\right) A\left[{}_{2}M_{sm}\left[\frac{{}_{2}N_{tn}}{(\gamma_t b)(\gamma_n b)}\right]\right]$$

$$[L_{stmn}]_{23} = -\frac{\theta}{A}\left[{}_{1}M_{sm}\left[\frac{{}_{2}N_{tn}}{\gamma_t b}\right]\right] - \frac{SCg\theta}{A}\left[{}_{1}M_{sm}\left[\frac{{}_{2}N_{tn}}{\gamma_t b}\right]\right]$$

$$[L_{stmn}]_{24} = 0$$

$$[L_{stmn}]_{25} = -St\theta\left[{}_{1}M_{sm}\left[\frac{{}_{2}N_{tn}}{(\gamma_t b)(\gamma_n b)}\right]\right]$$

$$[L_{stmn}]_{33} = \frac{\theta^2}{A}\left[{}_{1}M_{sm}\left[{}_{1}N_{tn}\right]\right] + \frac{Sg}{A}\left[{}_{1}M_{sm}\left[{}_{2}N_{tn}\right]\right] + SCgA\left[{}_{2}M_{sm}\left[{}_{1}N_{tn}\right]\right]$$

$$[L_{stmn}]_{34} = SCtA\left[\frac{{}_2M_{sm}}{\beta_m \ell}\,[{}_1N_{tn}]\right]$$

$$[L_{stmn}]_{35} = St\left[{}_1M_{sm}\left[\frac{{}_2N_{tn}}{\gamma_n b}\right]\right]$$

$$[L_{stmn}]_{44} = A\left[(\beta_m \ell)^2\ {}_1M_{sm}[{}_1N_{tn}]\right] + \frac{SCt^2A}{g}\left[\frac{{}_2M_{sm}}{(\beta_s \ell)(\beta_m \ell)}\,[{}_1N_{tn}]\right]$$

$$+ \left(\frac{1-\nu}{2}\right)\frac{1}{A}\left[\frac{{}_2M_{sm}}{(\beta_s \ell)(\beta_m \ell)}\,[{}_2N_{tn}]\right] \qquad \begin{matrix}(31)\\ \text{(cont'd)}\end{matrix}$$

$$[L_{stmn}]_{45} = \left(\frac{1+\nu}{2}\right)\left[\frac{{}_2M_{sm}}{\beta_s \ell}\left[\frac{{}_2N_{tn}}{\gamma_n b}\right]\right]$$

$$[L_{stmn}]_{55} = \frac{1}{A}\left[{}_1M_{sm}\left[(\gamma_n b)^2\ {}_1N_{tn}\right]\right] + \frac{St^2A}{g}\left[{}_1M_{sm}\left[\frac{{}_2N_{tn}}{(\gamma_t b)(\gamma_n b)}\right]\right]$$

$$+ \left(\frac{1-\nu}{2}\right) A\left[{}_2M_{sm}\left[\frac{{}_2N_{tn}}{(\gamma_t b)(\gamma_n b)}\right]\right]$$

The matrix nomenclature used in Equation (31) is somewhat unconventional and, therefore, is defined below. The matrix $[M_{sm}[N_{tn}]]$ in expanded form is

$$[M_{sm}[N_{tn}]] = \begin{bmatrix} M_{11}[N_{tn}] & M_{12}[N_{tn}] & \cdots\cdots & M_{1m}[N_{tn}] \\ M_{21}[N_{tn}] & M_{22}[N_{tn}] & \cdots\cdots & M_{2m}[N_{tn}] \\ \vdots & \vdots & & \vdots \\ M_{s1}[N_{tn}] & M_{s2}[N_{tn}] & \cdots\cdots & M_{sm}[N_{tn}] \end{bmatrix} \qquad (32)$$

This nomenclature has been introduced because it seems natural for this problem. The requirement arises in the integration of mode functions such as

$$\int_0^{\ell} \int_0^{b} \{X_s Y_t\} \lfloor X_m Y_n \rfloor \, dy \, dx \ . \tag{33}$$

Kinetic Energy

The general form of kinetic energy is indicated in Equation (13). After substitution of Equation (14) into Equation (13), and introducing the simplifications used in the strain energy, the resulting kinetic energy is:

$$T = \rho_2 h_2 \ell b \left\lfloor \lfloor q_{st} \rfloor_{\ell} \right\rfloor \left[[M_{stmn}]_{\ell k} \right] \left\{ \{q_{mn}\}_k \right\} \tag{34}$$

The terms in the non-dimensional mass matrix, M, are

$$\begin{aligned}
[M_{stmn}]_{11} &= (1 + gH) \left[\frac{{}_2M_{sm}}{(\beta_s \ell)(\beta_m \ell)} [{}_1N_{tn}] \right] \\
[M_{stmn}]_{22} &= (1 + gH) \left[{}_1M_{sm} \left[\frac{{}_2N_{tn}}{(\gamma_t b)(\gamma_n b)} \right] \right] \\
[M_{stmn}]_{33} &= (1 + gH) \left[{}_1M_{sm} [{}_1N_{tn}] \right] \\
[M_{stmn}]_{44} &= \left(1 + \frac{gH}{3}\right) \left[\frac{{}_2M_{sm}}{(\beta_s \ell)(\beta_m \ell)} [{}_1N_{tn}] \right] \\
[M_{stmn}]_{55} &= \left(1 + \frac{gH}{3}\right) \left[{}_1M_{sm} \left[\frac{{}_2N_{tn}}{(\gamma_t b)(\gamma_n b)} \right] \right]
\end{aligned} \tag{35}$$

One additional non-dimensional parameter has been introduced. This parameter is the mass ratio

$$H = \rho_1/\rho_2 \; .$$

Frequency Analysis

Lagrange's equation may be used to find the frequencies of free vibration. The following form is used in a free vibration analysis since the system is conservative:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_{st\ell}}\right) - \frac{\partial T}{\partial q_{st\ell}} + \frac{\partial U}{\partial q_{st\ell}} = 0 \tag{36}$$

The equations which result form an eigenvalue problem of the following form:

$$\left[[L_{stmn\ell k}] - \Omega^2 [M_{stmn\ell k}] \right] \{q_{mnk}\} = 0 \tag{37}$$

where

$$\Omega^2 = \frac{\rho_2 \ell b(1 - \nu^2)}{E} \omega^2$$

The system of equations expressed by Equation (37) must ultimately be reduced to a form suitable for computer solution. The multiple subscript, stmnℓk, must be replaced with a double subscript, ij, in the formation of the eigenvalue matrix. This change in subscripts is really only a matter of bookkeeping. Once accomplished, the problem becomes

$$[L_{ij}]\ \{q_j\} - \Omega^2 \lceil M_{ij} \rfloor\ \{q_j\} = 0 \tag{38}$$

The eigenvalue solution to be used is a method Jacobi developed. In effect, the solution requires diagonalization of the eigenvalue determinant by coordinate transformations. This method requires symmetry of the eigenvalue matrix. If, in Equation (38), the symmetric stiffness matrix is premultiplied by the inverse of the diagonal mass matrix, a non-symmetric eigenvalue matrix results. This problem is circumvented by a transformation. Let the diagonal matrix $\lceil M \rfloor$ be expressed as:

$$\lceil M_{ij} \rfloor = \lceil \sqrt{M_{ij}} \rfloor\ \lceil \sqrt{M_{ij}} \rfloor \tag{39}$$

Substitution into Equation (38) gives

$$[L_{ij}]\ \{q_j\} - \Omega^2 \lceil \sqrt{M_{ij}} \rfloor\ \lceil \sqrt{M_{ij}} \rfloor\ \{q_j\} = 0\ . \tag{40}$$

Premultiplying by the transpose of $\lceil \sqrt{M} \rfloor$ results in

$$\lceil \sqrt{M_{ij}} \rfloor^{-1} [L_{ij}]\ \lceil \sqrt{M_{ij}} \rfloor^{-1} \lceil \sqrt{M_{ij}} \rfloor\ \{q_j\} - \Omega^2 \lceil \sqrt{M_{ij}} \rfloor\ \{q_j\} = 0\ . \tag{41}$$

In this transformation, the mode shape becomes

$$\{x_i\} = \lceil\sqrt{M_{ii}}\rfloor \{q_i\} \ . \tag{42}$$

Since only the normalized eigenvectors are of interest, this substitution is permissable. Therefore, the eigenvalue problem assumes the final form of

$$\left[\frac{L_{ij}}{\sqrt{M_{ii}M_{jj}}}\right]\{x_i\} - \Omega^2 \lceil I \rfloor \{x_i\} = 0 \ . \tag{43}$$

CHAPTER III

NUMERICAL STUDY

An analysis of the type presented here is difficult to perform because of the large number of mechanical operations which must be made in the derivation. For this reason, it is desirable to use existing analytical methods to check numerical results, if they are applicable.

From the outset, the equations were arranged to allow confirmation of the final results with Freudenthal's analysis[3] for simply supported edges. The clamped beam functions were chosen because they satisfy the geometric boundary conditions and also have the same general form as Equation (14) for the displacements in terms of mode shapes. Use of identical generalized forms for the clamped and simply supported deflection series meant that the analysis could be conducted in one operation for both sets of mode functions. Also, the final equations would be applicable to both boundary conditions by changing only the values of the mode-function integrals, M_{sm} and N_{tn}.

Once the analysis was programmed, the results from the simply supported case were compared with results from Freudenthal's analysis. These results were found to be identical, as required.

This comparison also helped in the analysis where sine functions were used as the clamped-beam mode shapes, because of the similarity of the analyses.

After numerical confirmation of the digital analyses, three studies were made. Those studies were to determine

(1) if convergence could be obtained for the first mode frequency with

clamped edges,

(2) the amount of coupling between modes, and

(3) the effect of the various parameters on the modes.

In the accomplishment of these items, the values of the parameters used were for the experimental test panel. This panel is typical of construction in the aircraft industry.

Convergence Analysis

In determining the number of series terms required for convergence of the first mode frequency, several different combinations of modal series terms were used. The combinations and frequencies are listed in Table 1 for the beam function analysis. Examination of the tabulation shows that the fundamental frequency continues to decrease as the total number of terms increases. There is no assurance that convergence will occur since the reduction in frequency is 2.4% if the number of terms in each direction is increased from 1 to 2 and the decrease is 4.3% if the number of terms is increased from 2 to 3 in each direction.

Since the sine functions form a complete sequence, the chances for convergence using this set of assumed mode functions is better. Table II shows the effect of number of terms on calculated frequency. It is obvious that convergence is better using this series, since 9 terms (3, 3) decrease the first mode frequency only 0.75% below the 1 term value. Both these sequences were truncated because of computer limitations. The size of the eigenvalue matrix is 5 x m x n and is limited to 45.

The conclusions which can be made from the convergence analysis are (1) the beam function analysis does not show any definite trend toward convergence with 9 terms, (2) the sine function analysis appears to be converging in the first

Table 1. Effect of Number of Modal Series Terms on Convergence of Modal Frequency Using Beam Functions

No. of Terms		Non-dimensionalized Frequency $\Omega(1,1)$
Straight Edge M	Curved Edge N	
1	1	0.05732
1	2	0.05635
1	3	0.05411
2	1	0.05530
2	2	0.05594
2	3	0.05361
3	1	0.05674
3	2	0.05576
3	3	0.05339

Table 2. Effect of Number of Modal Series Terms on Convergence of (1,1) Mode Frequency Using Sine Functions

No. of Terms		Non-dimensionalized Frequency $\Omega(1,1)$
Straight Edge M	Curved Edge N	
1	1	0.07091
1	2	0.07083
1	6	0.07080
1	9	0.07080
2	2	0.07049
2	4	0.07046
3	3	0.07039
4	2	0.07039
9	1	0.07050

mode using 9 terms, and (3) the two analyses appear to be converging to different eigenvalues. The difference in frequency cannot be accounted for, since both analyses have been thoroughly checked. It is felt that the sine series probably gives more accurate results since it meets all the requirements of the Rayleigh-Ritz analysis; however, the beam function frequencies are more acceptable, since they are lower.

Modal Coupling Effects

The mode shape for each eigenvalue contains contributions from all the generalized coordinates unless the mode is a normal mode. The mode number is determined by the generalized coordinate with the greatest magnitude.

The computer output containing the eigenvalues and eigenvectors calculated for the test panel is given as Figure II.1 in Appendix II. The output for the sine modes is shown first. There is hardly any coupling in the flexural modes. For example, the flexural deflection shape for the (1,1) mode is approximately

$$w = 0.9992 \operatorname{Sin}\frac{\pi x}{\ell} \operatorname{Sin}\frac{\pi y}{b} - 0.0295 \operatorname{Sin}\frac{\pi x}{\ell} \operatorname{Sin}\frac{3\pi y}{b} - 0.0201 \operatorname{Sin}\frac{3\pi x}{\ell} \operatorname{Sin}\frac{3\pi y}{b}$$

The other terms are negligible.

Some of the in-plane and rotational modes are more highly coupled, as can be seen by examination of the mode shapes. However, these modes are not of primary concern in this analysis.

The mode shapes for the beam function analysis are more highly coupled as can be seen in Figure II.2. For example, the (1,3) mode shape for flexural deflections is

$$w = -0.0595X_1(x)Y_1(y) + 0.9858X_1(x)Y_3(y) + 0.1408X_1(x)Y_5(y) + 0.0555X_3(x)Y_3(y) + 0.0178X_5(x)Y_3(y)$$

These results also indicate that the sine functions are better assumptions, since the mode shapes are purer.

Parameter Study

It is very helpful to have a general idea of the effects of the various parameters on the results. This is useful in two ways. First, it indicates, in general, which parameters are important and which ones might be neglected in future analyses. Second, it provides a means of explaining certain deviations from experimental data, if the results happen to be particularly sensitive to a given parameter.

The experimental test panel was chosen as a basis of comparison for variation of the parameters. Each parameter was varied independently with all the others fixed at the text panel values.

The data shown were obtained from the beam function analysis. Enough data points were calculated with the sine function - clamped edge analysis to confirm that the trends were the same, although the calculated eigenvalues were larger for the lower modes.

Effect of Subtended Angle

The range chosen for this variation was 0 to 1 radian. As expected, an increase in curvature increases natural frequency of the panel as shown in Figure 3.

A point of interest is the tendency of the frequency of the (1, 1) mode to approach the frequency of the (1, 3) mode. This effect is caused by a buildup of stretching energy in the lower modes as curvature increases. This effect has been

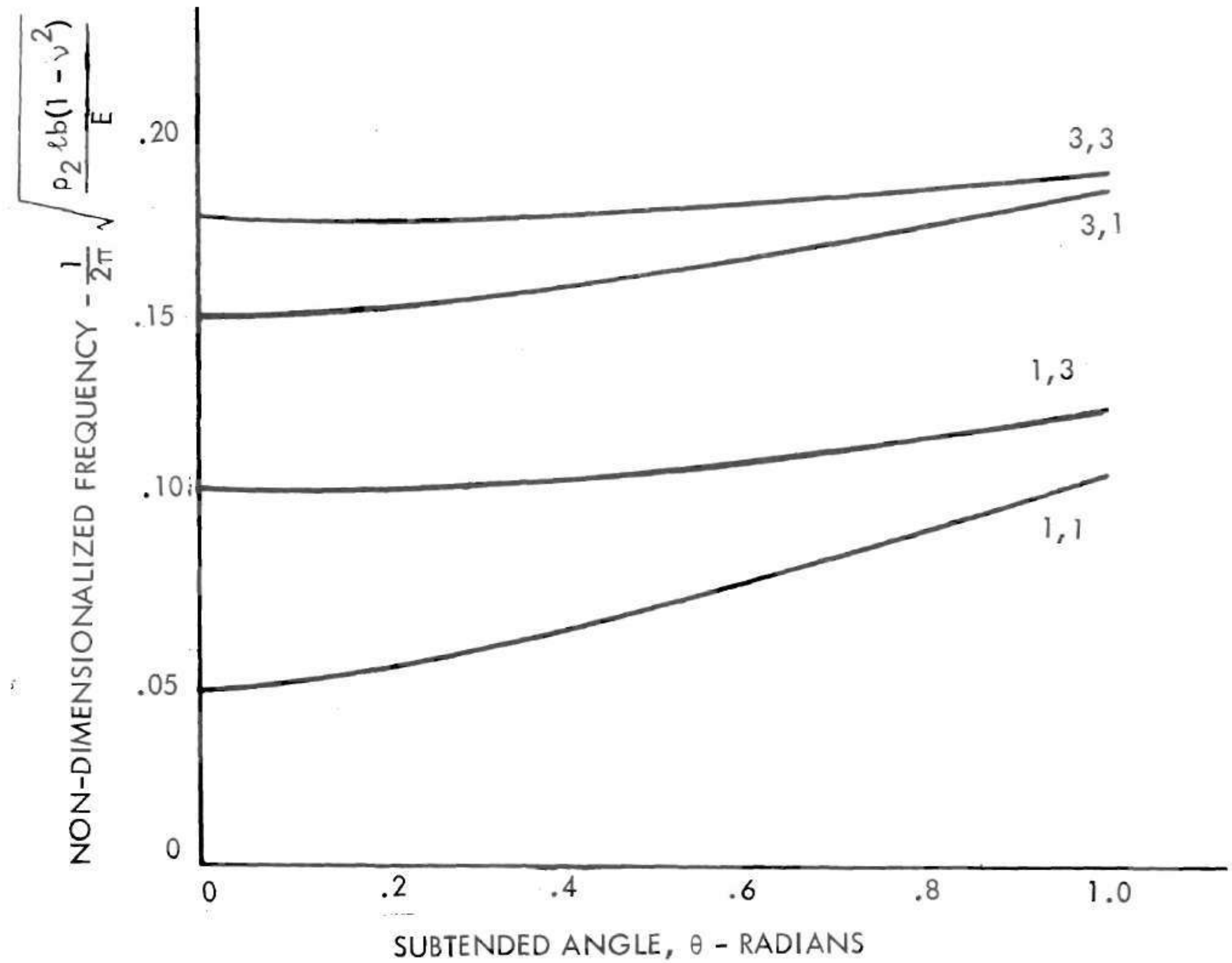

Figure 3. Effect of Panel Curvature on Characteristic Values

discussed in several papers, Arnold and Warburton[8] being the first to point out the effect. Scruggs[9], in his Master's Thesis, compared the stretching energy with bending energy for a cantilevered cylindrical shell. In this analysis it was shown that the minimum in strain energy is not in the (1,1) mode, as in a flat panel, but occurs in some higher mode of vibration. The mode with the minimum ratio of strain energy to generalized mass is generally the lowest frequency mode.

Effect of Core to Skin Thickness Ratio

With all other parameters fixed, an increase in the core to skin thickness ratio can be looked at more simply as a core thickness increase with all remaining panel dimensions fixed. The obvious effect is one of increasing the bending stiffness of the panel. For relatively thick cores the flexural rigidity of a simple sandwich plate reduces to

$$D \approx 2E \frac{h_1^2 h_2}{(1 - \nu^2)} \qquad h_1 \gg h_2$$

For a flat honeycomb panel, Sweers[10] shows that the first mode natural frequency is proportional to $\sqrt{D}$, if shear modulus effects can be neglected. Therefore, frequency would be proportional to core thickness for large core to skin thickness ratios. The frequency increase experienced in Figure 4 is closer to a square-root function of core thickness. The reasons for the difference between this and Sweer's analysis is the inclusion of core shear modulus and the radius of curvature of the panel.

Effect of Core Density/Skin Density Ratio

The value of core/skin density ratio for practical aircraft construction makes this an almost negligible quantity, and it could be discarded unless analyses

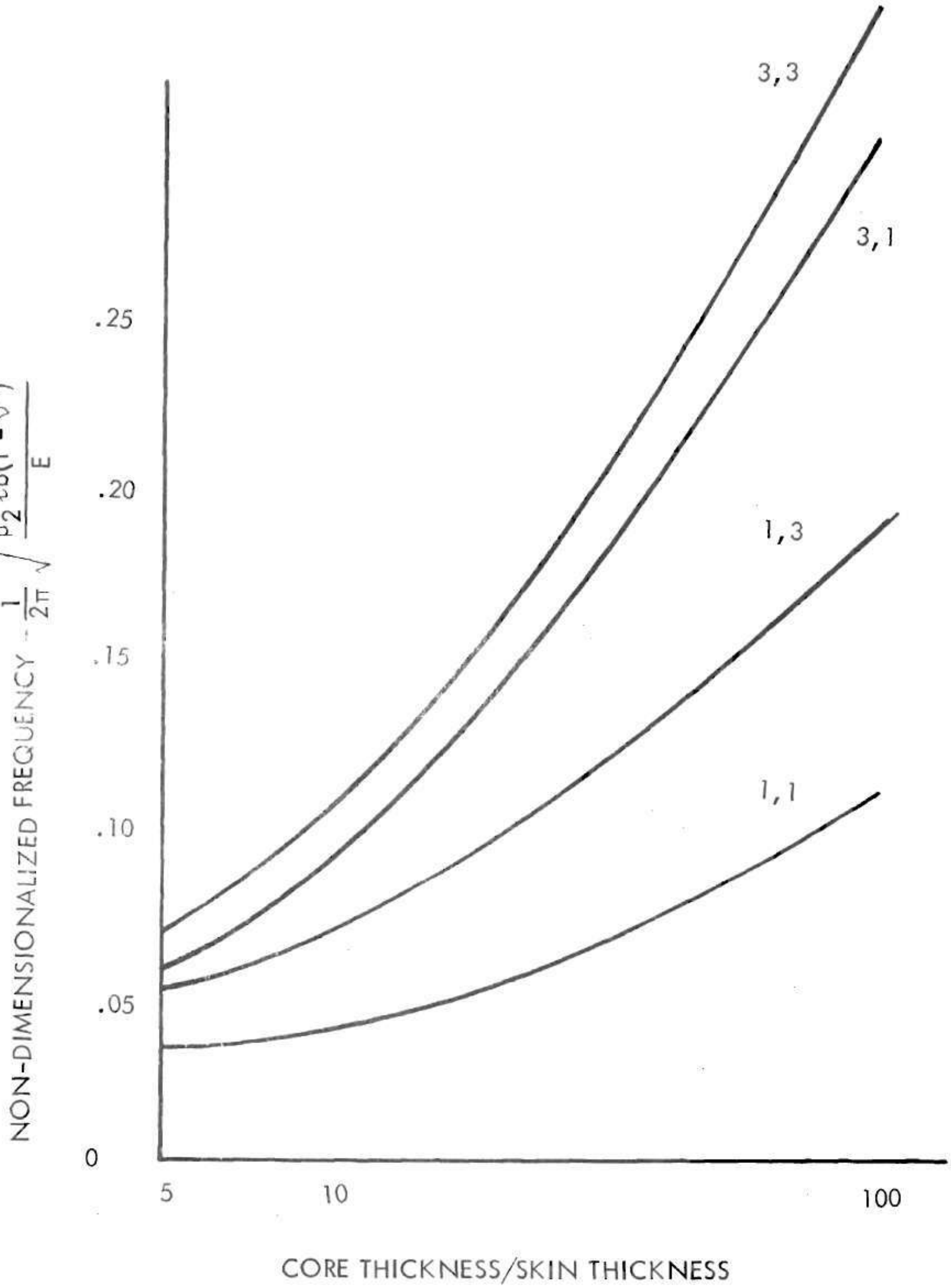

Figure 4. Effect of Core/Skin Thickness Ratio on Characteristic Values

with heavier core materials were contemplated. The effect of increasing the ratio is predictable. Examining Figure 5 it can be seen that an asymptote is approached for values less than 0.01. Then as the ratio is increased - the mass increases, the spring does not change - frequency decreases.

Effect of Ratio of Shear Modulus of the Core to Young's Modulus of the Skin

This parameter variation emphasizes the requirement of including transverse shear in the sandwich panel analysis. The effect of varying the ratio is of considerable magnitude (see Figure 6). The consideration of transverse shear effectively increases flexibility of a solid plate. For a sandwich plate with a weak core, the flexibility is greatly increased. From simple beam theory, these effects can be more readily perceived. The stiffness correction factor for a honeycomb beam is

$$\frac{1}{1+\beta}$$

where

$$\beta = \frac{\pi^2 E h_1 h_2}{G \ell^2 (1 - \nu^2)} = \frac{\pi^2 g}{S t^2}$$

From this relationship, it can be observed that as the ratio of G/E is reduced the stiffness factor increases and natural frequency would be expected to decrease. For the basic panel, $g \approx 12$ and $t \approx 1000$ so that $\beta \approx 1.2 \times 10^{-4}/S$. Therefore, for values of $S \leq 5 \times 10^{-4}$ a significant change in flexural rigidity can be expected.

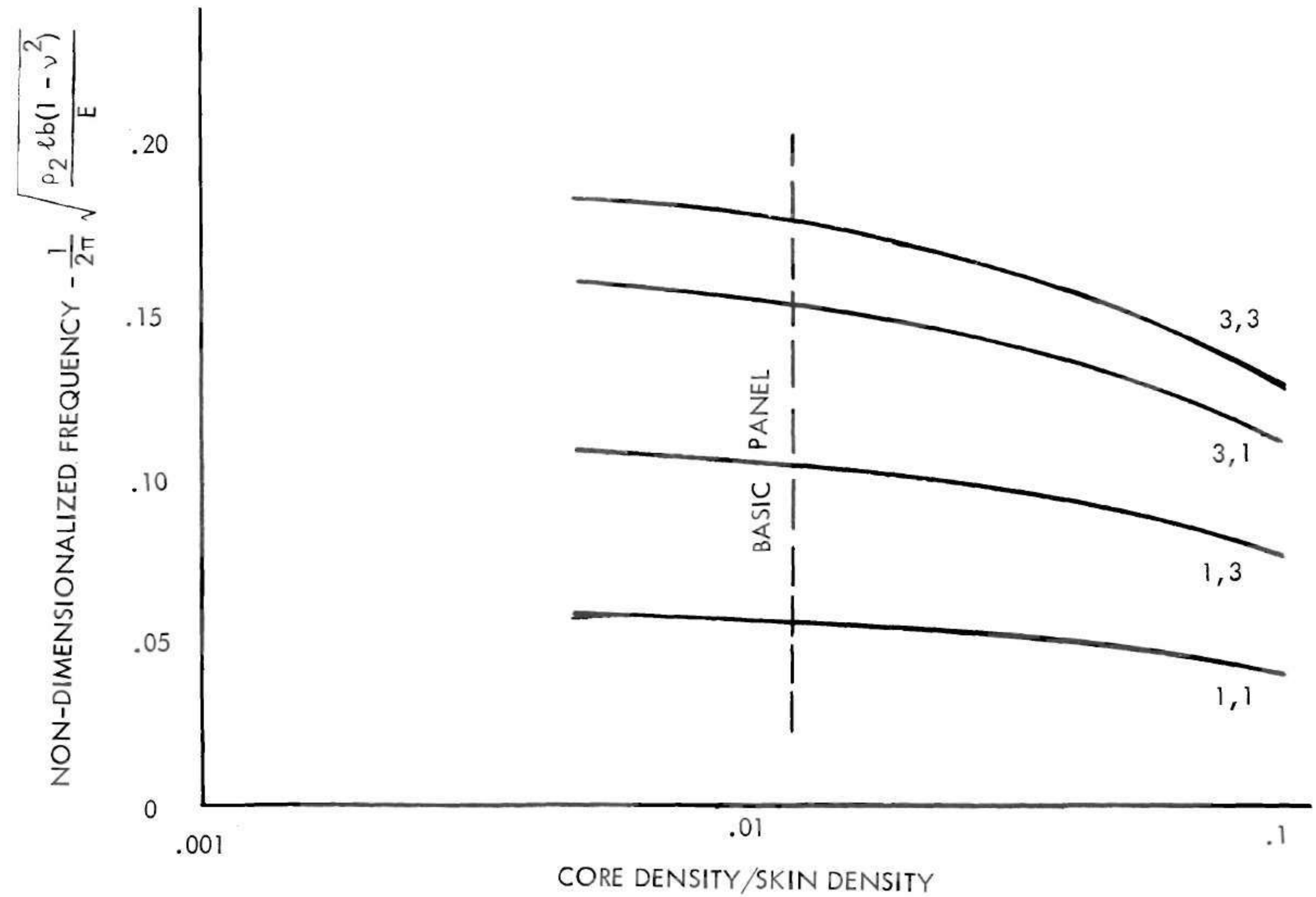

Figure 5. Effect of Core/Skin Density Ratio on Characteristic Values

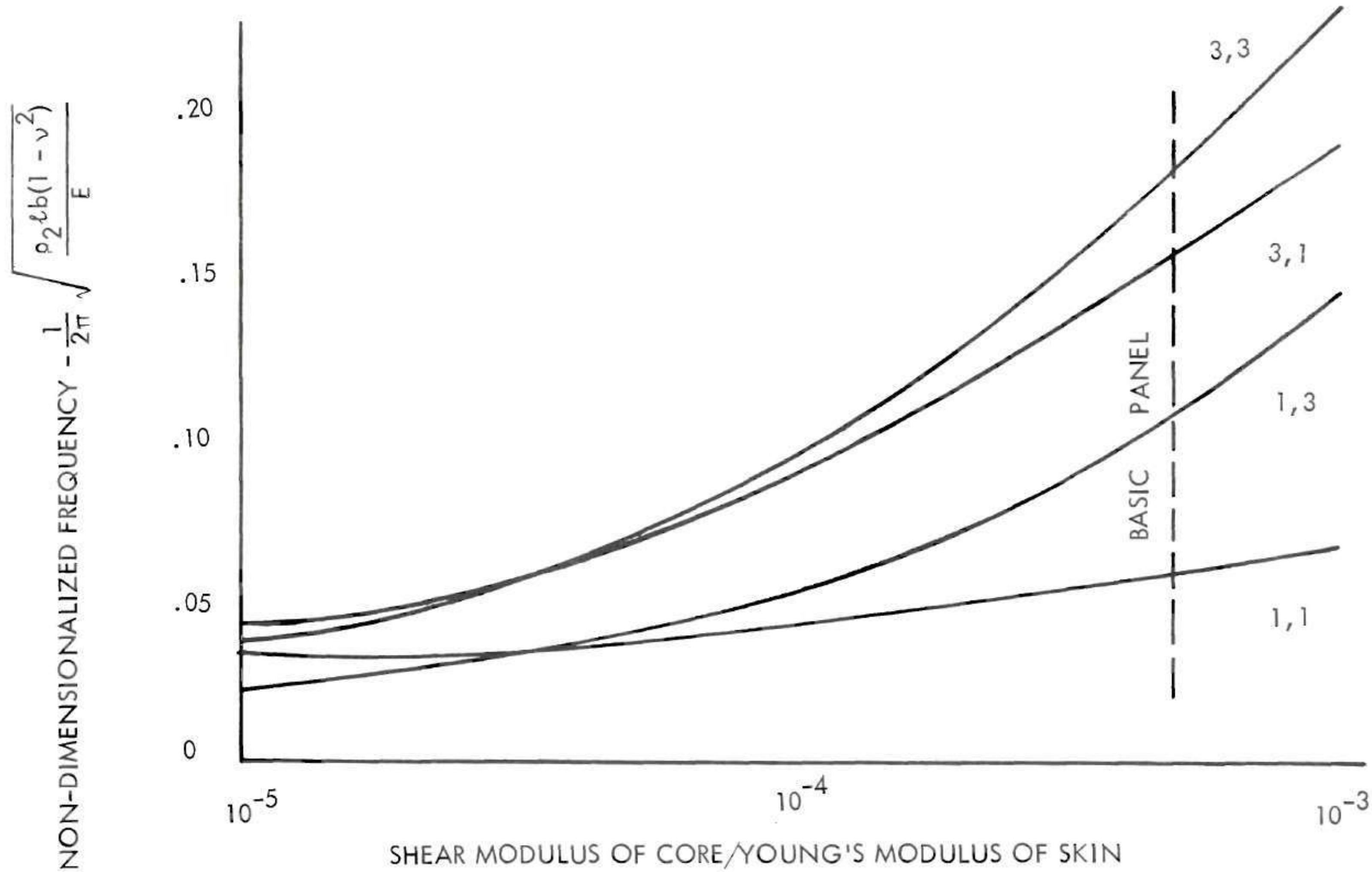

Figure 6. Effect of the Ratio of Shear Modulus of the Core to Young's Modulus of the Skin on Eigenvalues

CHAPTER IV

EXPERIMENTAL PROCEDURE WITH THE COMPARISON OF CALCULATED AND MEASURED RESULTS

There are only three references available which comment on the problems that arise in attempting to effectively clamp the edges of curved panels. These studies[6,15,16] all indicate extreme difficulties in obtaining the complete clamped condition.

Reference 6 presents the most detailed set of experimental data that has been published for cylindrically curved panels. In this study, natural frequencies and Chladni patterns were found for most modes from the (1,1) through the (5,5) mode. Panels 9" x 11" of two thicknesses - 0.032" and 0.048" - were tested at four radii - 48", 72", 96", and ∞ (or flat). In some unpublished tests, one panel was selected to demonstrate the effect of clamping on natural frequency. This panel, 0.048" thick at 96" radius of curvature, was first attached to a rigid frame by bolting at one inch intervals around the perimeter of the panel. Next, a similar rigid frame was placed on the top surface of the panel. The frames, which sandwiched the edges of the panel, were fastened with bolts on 1" centers. The natural frequencies increased. Finally, a second row of bolts, staggered with respect to the first row, was included to further clamp the frame together. Increases in frequencies of as much as 12% were noted when compared to the values obtained from the first experiment described. Still, the (1,1) mode experimental frequency remained 29% below the calculated value. The calculated values of Reference 6 were confirmed by Sewall.[15]

Other experiments (unpublished) were run in conjunction with the tests of Reference 6 to determine the necessary requirements to provide a perfectly clamped edge for the simple curved panels. However, other problems were encountered. As the frames were made more massive and rigid and the panels were made thinner, thermal effects completely negated the tests. Of course, the thin panel reacted to temperature changes much more rapidly than the massive boundary clamp, which for practical purposes stayed at constant temperature for small variations in room temperature. Since the boundaries of the panel were very rigidly clamped, extension or shrinkage due to temperature changes was directly reflected as a pre-stress in the panel. In one instance, by changing the temperature 4°F with a heat lamp (measured with a thermocouple attached directly to the panel), about a 50% decrease of the first mode frequency resulted. The panel was flat and made of 0.020" thick aluminum 14" x 16" in size. The frame was made of 1/2" steel plate. The panel was sandwiched around its perimeter between two identical sections of the frame.

With the problems experienced in trying to clamp the edges of a simple flat or curved plate, it was felt that some other method should be attempted with the sandwich panel. Comparing the two panels, the plate stiffness of the sandwich panel is about 250 times greater than that of the simple panel tested in Reference 6.

The major reason for the difficulties experienced in clamping the edges of the curved panels is definitely related to the curvature, since no apparent problems, other than the temperature variation mentioned above, have been encountered in clamping flat beams or plates of simple[17] or laminated construction.[1,2]

The explanation seems to be that the in-plane motion and rotation in the curved direction, $\bar{v}$ and φ, are very difficult to restrain. When compared to w, $\bar{v}$ is normally 2 to 3 orders of magnitude smaller. A criteria for acceptable clamping is proposed and can be expressed by placing the following restrictions on the deflections:

$$w_{edge} \leq \varepsilon\, w_{max}$$

$$\bar{u}_{edge} \leq \varepsilon\, \bar{u}_{max}$$

$$\bar{v}_{edge} \leq \varepsilon\, \bar{v}_{max}$$

$$\psi_{edge} \leq \varepsilon\, \psi_{max}$$

$$\varphi_{edge} \leq \varepsilon\, \varphi_{max}$$

If $\bar{v}_{max}$ is 1000 times smaller than w_{max}, then v_{edge} must be 1000 times smaller than w_{edge}. This implies that the frame must be extremely rigid in the direction of in-plane motions.

Test Arrangement for Curved Sandwich Panel

For the experimental verification of the calculations, a sandwich panel was selected which was constructed for another series of tests.[6] The physical data and description of the panel are listed below:

ℓ - length	16.50"
b - arc length	23.00"
$2h_1$ - core thickness	0.372"
h_2 - skin thickness	0.016"
$_2\rho$ - core density	$5.25 \times 10^{-6}\ \frac{\#\text{-sec}^2}{\text{in}^4}$
$_1\rho$ - skin density	$4.15 \times 10^{-4}\ \frac{\#\text{-sec}^2}{\text{in}^4}$

Core material - Fiberglass Honeycomb

G_{xz}	1.80×10^4 psi
G_{yz}	9.05×10^3 psi

Skin Material - Titanium

E	1.62×10^7 psi
G	6.13×10^6 psi
ν	0.322

The edge mounting details are shown in Figure 7. The honeycomb core was removed approximately 0.75" from the edge and an epoxy potting compound was used to fill this void. This was done in an effort to prevent crushing of the panel at the edge and also to try to prevent core edge rotation. In a further effort to prevent edge slippage and rotation, the perimeter of the panel was drilled for 0.25" bolts on 1" centers. Studs 2.5" long were then installed in the perimeter mounting holes.

The panel was placed in a plywood frame and molten Cerrobend was cast around the panel edge shown in Figure 7. A photograph of the panel-frame combination is shown in Figure 8.

The Cerrobend was chosen because it has some rather peculiar characteristics for a metal. It melts at 158°F. When it cools it expands slightly. However, the modulus of elasticity is only 1.1×10^6 psi. After the test was begun, the Cerrobend exhibited an unusual elastic property. As the test was conducted, the edges of the panel became loose, causing the natural frequencies to decrease.

Experimental Procedure

The panel-frame combination was placed in a large test fixture built around an MB C-10, 1200# force vibration exciter. The setup is shown in Figure 9. The shaker force was input to the panel through a 1" thick rubber pad. The pad was bonded to the panel and to the shaker rod. The shaker rod attachment is shown in Figure 10.

Natural frequencies were detected by monitoring the acceleration of the

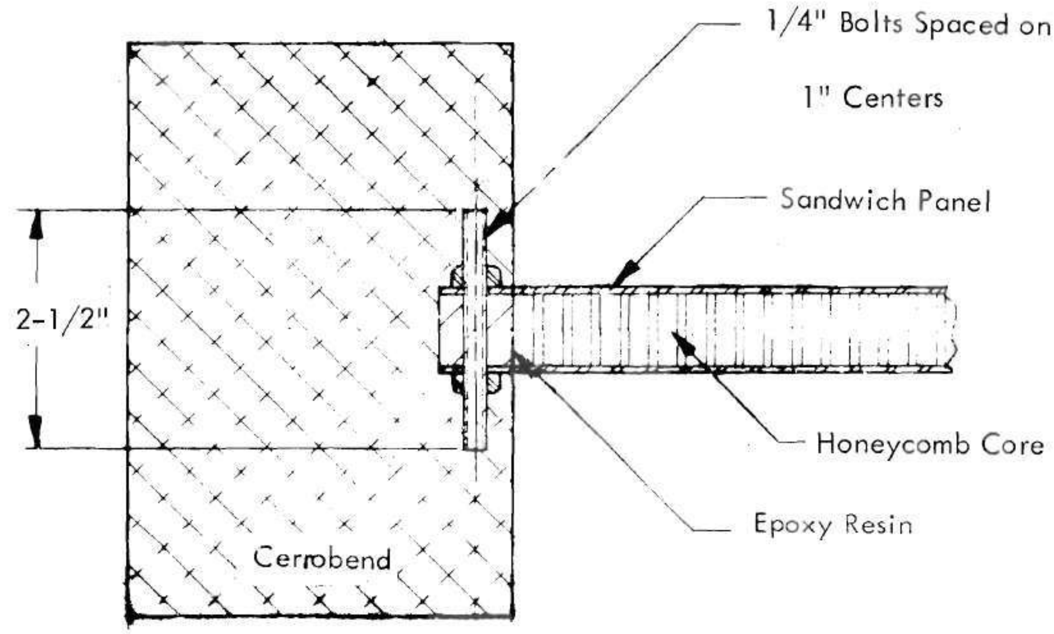

Figure 7. Details of Edge Construction

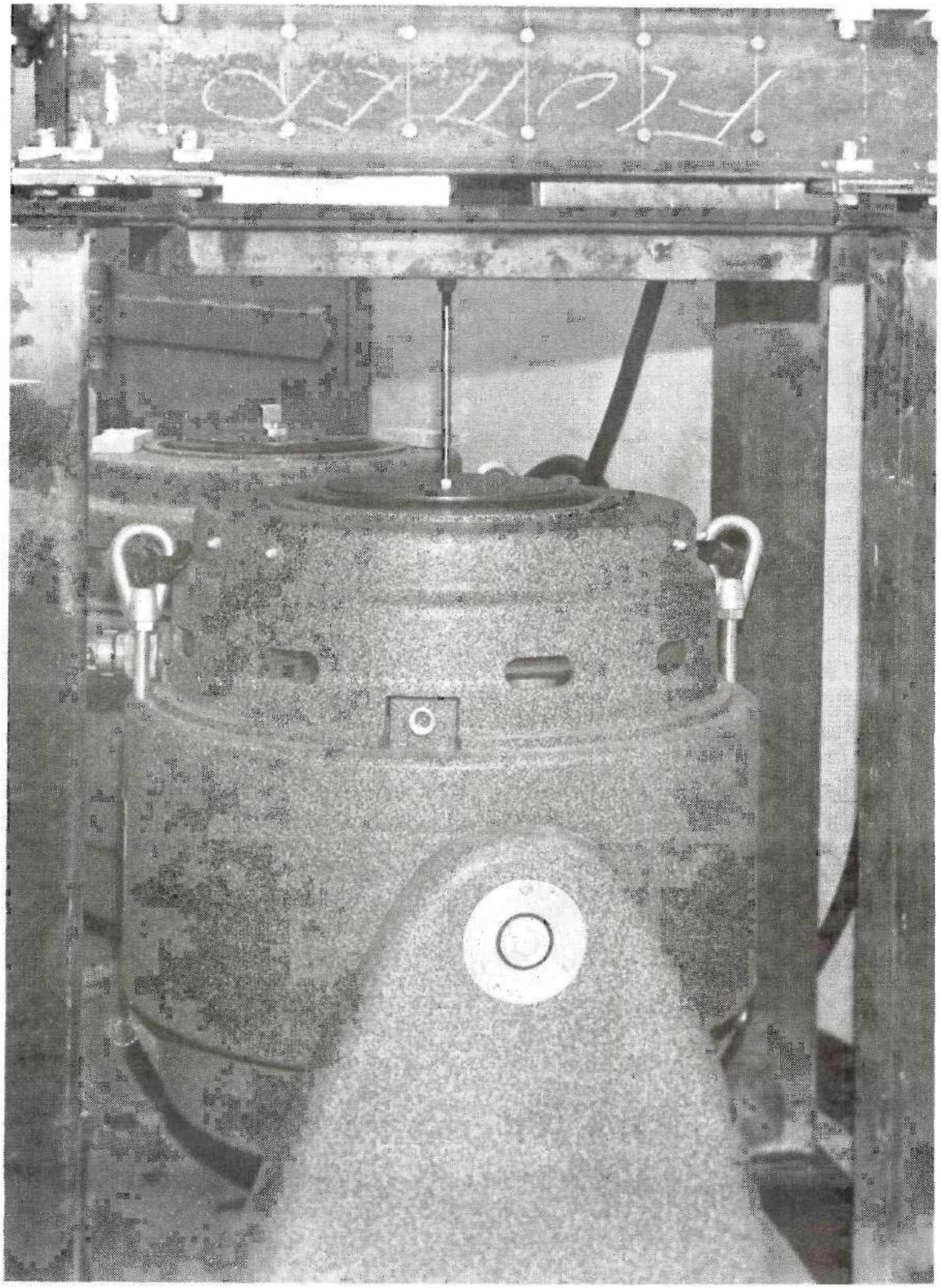

Figure 8. Test Arrangement

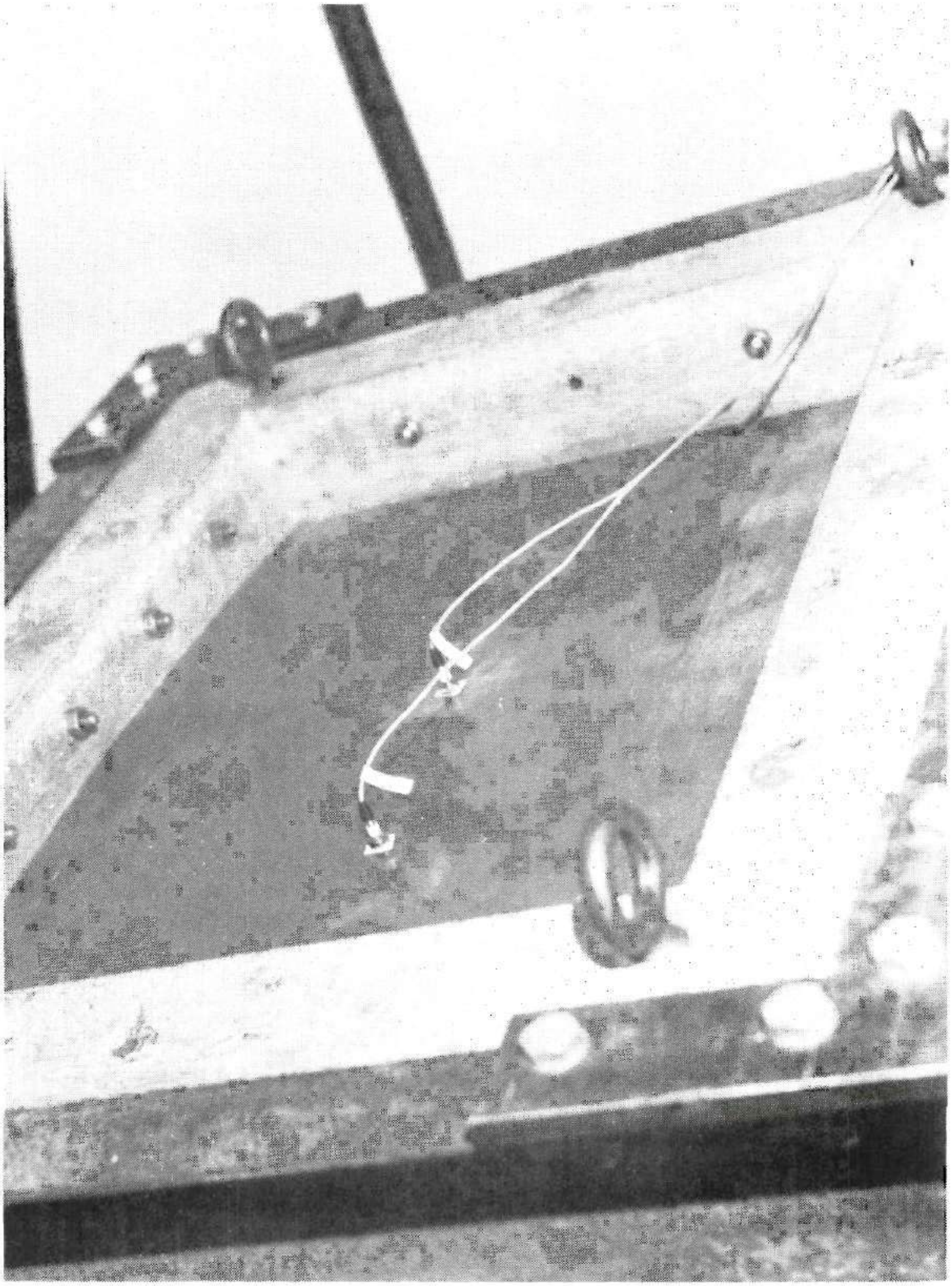

Figure 9. Panel/Frame Assembly

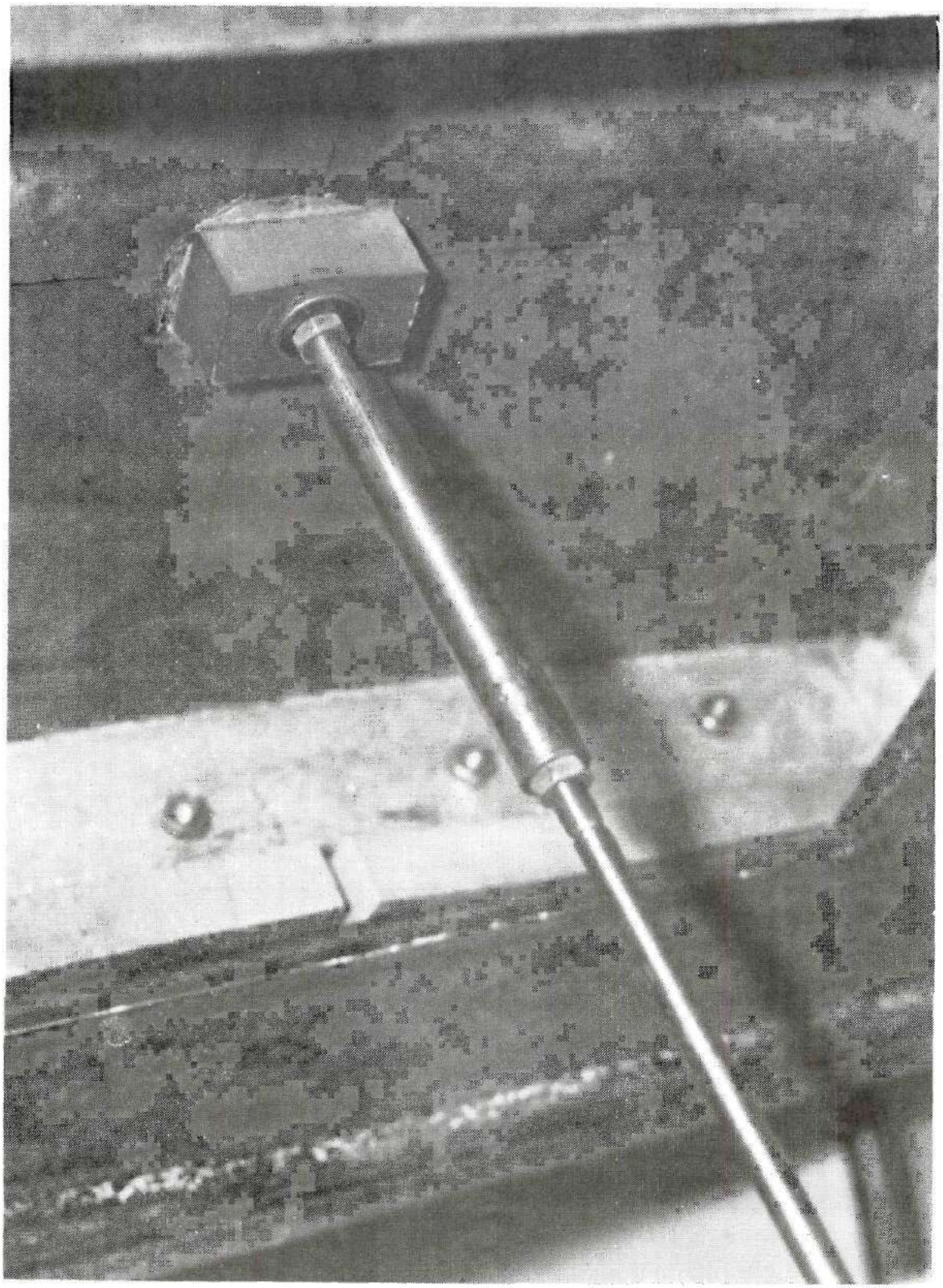

Figure 10. Shaker Attachment

panel during a constant force sine-sweep. The panel was sprinkled with cork dust to aid in the identification of mode shapes.

Because of the extreme stiffness of the panel and also because of the method of excitation, only 5 modes of vibration were detected.

Comparison of Calculations and Experimental Values

The measured values of natural frequency are tabulated in Table III. Values calculated for simply-supported edges and with both approximate methods for clamped edges are compared with the measured data.

Examination of the measured values reveals that the experimental data are not bounded by the calculated values. This disagreement of results is probably associated with the loosening of the edges of the panel as described in the previous section.

However, the differences can be explained by considering a variation of the shear modulus of the core. The shear modulus determination is discussed by Kelsey, Gellatly, and Clark.[18] Several different methods are used. Variations in the values of shear modulus of up to 50% are obtained by the different methods. It is also shown that the face thickness affects the actual value of core shear modulus. Therefore, it is very likely that published values of core shear modulus could be in error by as much as 50%.

A $\pm$50% variation in shear modulus would place the following limits on some of the eigenvalues calculated with the beam function analysis: (1) The (1,1) mode frequency would vary from 535 cps to 642 cps. (2) The (1,3) mode frequency would vary from 812 cps to 1328 cps. (3) The (3,1) mode would vary from 1305 cps to 1840 cps. (4) The (3,3) mode frequency would vary from 1455 cps to 2200 cps. With variation of this magnitude attributed to variation in shear modulus, it can be easily seen how the experimental values of frequency are not bracketed by the clamped and simply supported theory.

Table 3. Comparison of Calculated and Measured Natural Frequencies

No. of Panel Active Areas Along		Calculated Frequency in CPS			Measured Frequency in CPS
Straight Edge	Curved Edge	Clamped Sine Functions	Clamped Beam Functions	Simply Supported	
1	1	754	599	406	421
1	2	916	825	not calc.	640
1	3	1145	1099	1009	839
1	5	1712	1732	1660	1387
3	2	2080	1740	not calc.	1302

CHAPTER V

CONCLUSIONS

The Rayleigh-Ritz method has been used to analytically determine the free vibration characteristics of cylindrically curved sandwich panels with all edges clamped and all edges simply supported.

The solution for simply supported edges is an exact solution.

Two sets of mode shapes were used in determining the eigenvalues for clamped edges. The values obtained from the computer analysis did not agree for the two sets of modes.

The experimental data do not compare well with the calculated values. This disagreement is probably caused by one or both of the following reasons:

(1) The edges of the panel became loose during testing.

(2) The shear modulus values for the core are subject to error.

CHAPTER VI

RECOMMENDATIONS

The following recommendations are made:

Determine the requirements for an effective edge clamp for curved panels.

Determine better methods of measuring the shear modulus of a honeycomb sandwich core.

Conduct all panel vibration tests using acoustic excitation, rather than mechanical shakers.

Develop a curved sandwich panel with stiffness reduced by 1 or 2 orders of magnitude (as compared to the panel tested in this study). This "weaker" panel will be easier to clamp and natural frequencies of free vibration will be easier to experimentally determine.

APPENDIX I

The elements in the stiffness and mass matrix, resulting from the analysis which used sine mode functions to represent clamped edges, are listed in this appendix. The functions ${}_1M_{sm}$, ${}_2M_{sm}$, ${}_1N_{tn}$, and ${}_2N_{tn}$ are given in Equation (29).

$$[L_{stmn}]_{11} = A\left[(\beta_m \ell)^2 {}_1M_{sm}[{}_1N_{tn}]\right] + \frac{1}{A}\left(\frac{1-\nu}{2}\right)\left[{}_1M_{sm}[(\gamma_n b)^2 {}_1N_{tn}]\right]$$

$$[L_{stmn}]_{12} = -\left(\frac{1+\nu}{2}\right)\left[{}_2M_{sm}[{}_2N_{tn}]\right]$$

$$[L_{stmn}]_{13} = -\nu\theta\left[{}_2M_{sm}[{}_1N_{tn}]\right]$$

$$[L_{stmn}]_{14} = 0$$

$$[L_{stmn}]_{15} = 0$$

$$[L_{stmn}]_{22} = \frac{1}{A}\left[{}_1M_{sm}[(\gamma_n b)^2 {}_1N_{tn}]\right] + \frac{Sg\theta^2}{A}\left[{}_1M_{sm}[{}_1N_{tn}]\right] + \left(\frac{1-\nu}{2}\right) A\left[(\beta_m \ell)^2 {}_1M_{sm}[{}_1N_{tn}]\right]$$

$$[L_{stmn}]_{23} = \frac{\theta}{A}(1 + SCg)\left[{}_1M_{sm}[{}_2N_{tn}]\right]$$

$$[L_{stmn}]_{24} = 0$$

$$[L_{stmn}]_{25} = -St\theta\left[{}_1M_{sm}[{}_1N_{tn}]\right]$$

$$[L_{stmn}]_{33} = \frac{\theta^2}{A}\left[{}_1M_{sm}[{}_1N_{tn}]\right] + \frac{Sg}{A}\left[{}_1M_{sm}[(\gamma_n b)^2 {}_1N_{tn}]\right] + SCgA\left[(\beta_m \ell)^2 {}_1M_{sm}[{}_1N_{tn}]\right]$$

$$[L_{stmn}]_{34} = SCtA\left[{}_2M_{sm}[{}_1N_{tn}]\right]$$

$$[L_{stmn}]_{35} = St[{}_1M_{sm}[{}_2N_{tn}]]$$

$$[L_{stmn}]_{44} = A[(\beta_m \ell)^2 {}_1M_{sm}[{}_1N_{tn}]] + \frac{SCt^2A}{g}[{}_1M_{sm}[{}_1N_{tn}]]$$

$$+ \frac{1}{A}\left(\frac{1-\nu}{2}\right)[{}_1M_{sm}[(\gamma_n b)^2 {}_1N_{tn}]]$$

$$[L_{stmn}]_{45} = -\left(\frac{1+\nu}{2}\right)[{}_2M_{sm}[{}_2N_{tn}]]$$

$$[L_{stmn}]_{55} = \frac{1}{A}[{}_1M_{sm}[(\gamma_n b)^2 {}_1N_{tn}]] + \frac{St^2A}{g}[{}_1M_{sm}[{}_1N_{tn}]]$$

$$+ A\left(\frac{1-\nu}{2}\right)[(\beta_m \ell)^2 {}_1M_{sm}[{}_1N_{tn}]]$$

Elements below the diagonal are not included since the stiffness matrix is symmetrical.

The mass matrix elements are:

$$[M_{stmn}]_{11} = (1 + gH)[{}_1M_{sm}]_1N_{tn}]]$$

$$[M_{stmn}]_{22} = (1 + gH)[{}_1M_{sm}[{}_1N_{tn}]]$$

$$[M_{stmn}]_{33} = (1 + gH)[{}_1M_{sm}[{}_1N_{tn}]]$$

$$[M_{stmn}]_{44} = (1 + gH/3)[{}_1M_{sm}[{}_1N_{tn}]]$$

$$[M_{stmn}]_{55} = (1 + gH/3)[{}_1M_{sm}[{}_1N_{tn}]]$$

All off diagonal elements are zero.

APPENDIX II

The computer results referred to in Chapter III are listed in this section. The first set gives eigenvalues for the test panel using sine functions to satisfy the clamped edge conditions. The second set of computer output utilized the beam functions to approximate the clamped edge boundaries.

DIMENSIONLESS FREQUENCIES AND NORMALIZED EIGENVECTORS
FOR
A CYLINDRICALLY CURVED SANDWICH PANEL
WITH
CLAMPED EDGES

****

NONDIMENSIONAL INPUT PARAMETERS

SUBTENDED ANGLE = 0.2744 ASPECT RATIO = 1.3935
LENGTH/SKIN THICKNESS = 1031.25 CORE/SKIN THICKNESS RATIO = 23.250
CORE/SKIN DENSITY RATIO = .0124740 POISSONS RATIO = .322
RATIO OF CORE TRANSVERSE SHEAR MODULI(GXZ/GYZ) = 1.98855
CORE SHEAR MODULUS/SKIN YOUNGS MODULUS = .00050071

NUMBER OF SERIES TERMS ALONG STRAIGHT EDGE = 3, ALONG CURVED EDGE = 3

****

COMPUTED FREQUENCIES AND MODE SHAPES

FREQUENCY =	0.7035D-01	0.1071D 00	0.1601D 00	0.1745D 00	0.1939D 00
GEN COORD	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE
U(1,1)	0.2233D-02	-0.1855D-03	-0.1514D-03	-0.3882D-02	0.1388D-03
U(1,3)	-0.2582D-03	0.2279D-02	-0.3580D-04	0.1930D-03	-0.3513D-02
U(1,5)	-0.9268D-04	-0.8262D-04	0.1561D-02	0.5080D-04	0.1567D-03
U(3,1)	0.2122D-03	-0.1795D-04	-0.1606D-04	0.1465D-02	-0.5045D-04
U(3,3)	-0.2938D-04	0.2589D-03	-0.4516D-05	-0.8804D-04	0.1623D-02
U(3,5)	-0.1350D-04	-0.1136D-04	0.2353D-03	-0.5308D-04	-0.9301D-04
U(5,1)	0.5515D-04	-0.5144D-05	-0.4315D-05	0.2327D-03	-0.8602D-05
U(5,3)	-0.8405D-05	0.7105D-04	-0.9065D-06	-0.1521D-04	0.2716D-03
U(5,5)	-0.4090D-05	-0.2936D-05	0.6949D-04	-0.9781D-05	-0.1671D-04
V(1,1)	-0.8647D-02	0.1447D-01	0.6173D-02	-0.3027D-04	0.6477D-04
V(1,3)	-0.7512D-03	-0.6019D-02	0.7509D-02	-0.3741D-06	-0.1552D-04
V(1,5)	-0.2011D-03	-0.9663D-03	-0.4248D-02	-0.7315D-05	-0.8889D-05
V(3,1)	0.1524D-03	-0.2171D-03	-0.6533D-04	-0.4158D-02	0.7187D-02
V(3,3)	0.1907D-04	0.1430D-03	-0.1256D-03	-0.6825D-03	-0.4748D-02
V(3,5)	0.4227D-05	0.2610D-04	0.8253D-04	-0.2059D-03	-0.8857D-03
V(5,1)	0.3278D-04	-0.4980D-04	-0.1610D-04	0.4221D-04	-0.7640D-04
V(5,3)	0.5129D-05	0.4561D-04	-0.4439D-04	0.9619D-05	0.7197D-04
V(5,5)	0.7102D-06	0.9405D-05	0.3337D-04	0.2654D-05	0.1490D-04
W(1,1)	$\boxed{0.9992D\ 00}$	0.2952D-01	0.8077D-02	0.2021D-01	0.4013D-03
W(1,3)	-0.2947D-01	$\boxed{0.9990D\ 00}$	0.1727D-01	0.3064D-04	0.1686D-01
W(1,5)	-0.7479D-02	-0.1753D-01	$\boxed{0.9995D\ 00}$	0.9024D-03	0.4529D-03
W(3,1)	-0.2012D-01	-0.8416D-03	-0.1134D-02	$\boxed{0.9952D\ 00}$	0.1386D-01
W(3,3)	0.3800D-03	-0.1680D-01	-0.9031D-03	-0.1387D-01	$\boxed{0.9952D\ 00}$
W(3,5)	0.4030D-04	0.9054D-04	-0.1284D-01	-0.4412D-02	-0.1342D-01
W(5,1)	-0.5840D-02	-0.2130D-03	-0.8874D-04	-0.1643D-01	-0.3259D-03
W(5,3)	0.1183D-03	-0.5141D-02	-0.1703D-03	0.1367D-03	-0.1528D-01
W(5,5)	0.7292D-05	0.1970D-04	-0.4250D-02	-0.1421D-05	0.3928D-04
PSI(1,1)	0.1072D-01	0.3736D-03	0.1565D-03	-0.1836D-01	-0.3080D-03
PSI(1,3)	-0.2291D-03	0.9423D-02	0.2579D-03	0.1611D-03	-0.1638D-01
PSI(1,5)	-0.2153D-04	-0.6277D-04	0.7740D-02	0.2448D-04	0.9000D-04
PSI(3,1)	0.1597D-02	0.6696D-04	0.1372D-04	0.1439D-01	0.2439D-03
PSI(3,3)	-0.4628D-04	0.1876D-02	0.4358D-04	-0.1442D-03	0.1338D-01
PSI(3,5)	-0.4910D-05	-0.1455D-04	0.1703D-02	-0.1273D-04	-0.7329D-04
PSI(5,1)	0.6465D-03	0.2209D-04	0.5968D-05	0.2860D-02	0.4837D-04
PSI(5,3)	-0.1501D-04	0.6238D-03	0.1564D-04	-0.2902D-04	0.2727D-02
PSI(5,5)	-0.1280D-05	-0.4081D-05	0.5556D-03	-0.1854D-05	-0.1416D-04
PHI(1,1)	0.7376D-02	-0.1231D-01	-0.5214D-02	0.2896D-03	-0.4456D-03
PHI(1,3)	0.1295D-02	0.9843D-02	-0.1233D-01	0.4908D-04	0.3356D-03
PHI(1,5)	0.4096D-03	0.1556D-02	0.8646D-02	0.2950D-04	0.7732D-04
PHI(3,1)	0.1620D-04	-0.2670D-04	-0.9618D-05	0.4666D-02	-0.8169D-02
PHI(3,3)	0.2973D-05	0.2350D-04	-0.2949D-04	0.1048D-02	0.7358D-02
PHI(3,5)	0.1251D-05	0.5265D-05	0.1826D-04	0.3732D-03	0.1603D-02
PHI(5,1)	0.4850D-04	-0.7359D-04	-0.2662D-04	0.7825D-04	-0.1291D-03
PHI(5,3)	0.1158D-04	0.7786D-04	-0.8222D-04	0.2041D-04	0.1340D-03
PHI(5,5)	0.4578D-05	0.1953D-04	0.7002D-04	0.8798D-05	0.3369D-04

Figure II-1. Computer Output for Sine Function-Clamped Panel

FREQUENCY =	0.2282D 00	0.2918D 00	0.3039D 00	0.3270D 00	0.8521D 00
GEN COORD	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE
U(1,1)	0.1485D-03	-0.1659D-02	0.2513D-04	0.3354D-04	-0.4816D-01
U(1,3)	0.1364D-03	0.3509D-04	-0.1384D-02	0.4678D-04	0.4905D-01
U(1,5)	-0.2557D-02	0.2034D-04	0.4721D-04	-0.6590D-03	0.1025D-01
U(3,1)	-0.5393D-04	-0.1811D-02	0.2560D-04	0.3638D-04	-0.2386D-02
U(3,3)	-0.6132D-04	0.4538D-04	-0.1883D-02	0.6558D-04	0.3873D-02
U(3,5)	0.1530D-02	0.3585D-04	0.8667D-04	-0.1798D-02	0.1319D-02
U(5,1)	-0.8996D-05	0.1002D-02	-0.1415D-04	-0.1986D-04	-0.6197D-03
U(5,3)	-0.1036D-04	-0.2635D-04	0.1088D-02	-0.3716D-04	0.1047D-02
U(5,5)	0.2767D-03	-0.2229D-04	-0.5346D-04	0.1118D-02	0.3960D-03
V(1,1)	0.3467D-04	0.3416D-04	-0.4651D-04	-0.8601D-05	0.9574D 00
V(1,3)	0.2899D-04	0.2999D-05	0.1927D-04	-0.1733D-04	-0.9861D-03
V(1,5)	-0.2025D-04	-0.1399D-06	0.1286D-05	0.7500D-05	-0.3850D-03
V(3,1)	0.2962D-02	-0.5140D-04	0.6961D-04	0.2015D-04	-0.7188D-02
V(3,3)	0.5949D-02	-0.8375D-05	-0.4047D-04	0.3304D-04	0.1130D-02
V(3,5)	-0.3838D-02	-0.4240D-05	-0.1070D-04	-0.2294D-04	0.5186D-03
V(5,1)	-0.3125D-04	-0.2003D-02	0.3540D-02	0.1446D-02	-0.1316D-02
V(5,3)	-0.8940D-04	-0.4837D-03	-0.3224D-02	0.4102D-02	0.3735D-03
V(5,5)	0.6818D-04	-0.1794D-03	-0.7275D-03	-0.3113D-02	0.2590D-03
W(1,1)	0.1551D-03	0.6289D-02	0.7769D-04	0.4545D-04	0.8246D-02
W(1,3)	0.3593D-03	0.1587D-04	0.5488D-02	0.1084D-03	-0.1488D-01
W(1,5)	0.1287D-01	0.4881D-04	0.5689D-04	0.4486D-02	-0.5970D-02
W(3,1)	0.4616D-02	0.1639D-01	0.1772D-03	0.1017D-03	-0.3489D-03
W(3,3)	0.1336D-01	0.1082D-04	0.1526D-01	0.2761D-03	0.4223D-03
W(3,5)	0.9994D 00	0.1233D-03	0.1419D-03	0.1364D-01	0.8408D-04
W(5,1)	-0.2341D-03	0.9995D 00	0.6151D-02	0.2471D-02	-0.1664D-03
W(5,3)	-0.4648D-03	-0.6163D-02	0.9995D 00	0.8792D-02	0.2038D-03
W(5,5)	-0.1363D-01	-0.2409D-02	-0.8796D-02	0.9996D 00	0.4653D-04
RSI(1,1)	-0.1453D-03	-0.7776D-02	-0.6277D-04	-0.3745D-04	0.9312D-04
RSI(1,3)	-0.3359D-03	0.2902D-04	-0.6933D-02	-0.1007D-03	-0.1418D-03
RSI(1,5)	-0.1360D-01	-0.3925D-06	0.1625D-04	-0.5727D-02	-0.4489D-04
RSI(3,1)	0.1163D-03	-0.1812D-01	-0.1368D-03	-0.7838D-04	0.1411D-04
RSI(3,3)	0.2769D-03	0.8166D-04	-0.1700D-01	-0.2282D-03	-0.2477D-04
RSI(3,5)	0.1190D-01	0.1116D-04	0.6631D-04	-0.1522D-01	-0.9404D-05
RSI(5,1)	0.2223D-04	0.1260D-01	0.9832D-04	0.5723D-04	0.2229D-05
RSI(5,3)	0.5599D-04	-0.5438D-04	0.1207D-01	0.1676D-03	-0.5618D-05
RSI(5,5)	0.2541D-02	-0.4196D-05	-0.4169D-04	0.1124D-01	-0.2769D-05
PHI(1,1)	-0.1468D-03	0.1962D-03	-0.3143D-03	-0.1093D-03	0.2623D-02
PHI(1,3)	-0.3357D-03	0.3964D-04	0.2359D-03	-0.2519D-03	-0.6908D-04
PHI(1,5)	0.2372D-03	0.1487D-04	0.5111D-04	0.1752D-03	-0.8696D-04
PHI(3,1)	-0.3444D-02	0.2681D-03	-0.4549D-03	-0.1713D-03	-0.1217D-04
PHI(3,3)	-0.9442D-02	0.6240D-04	0.3958D-03	-0.4616D-03	0.1764D-05
PHI(3,5)	0.7104D-02	0.2537D-04	0.9259D-04	0.3469D-03	0.3110D-06
PHI(5,1)	-0.4857D-04	0.2726D-02	-0.4869D-02	-0.2027D-02	-0.3975D-06
PHI(5,3)	-0.1534D-03	0.7449D-03	0.5020D-02	-0.6533D-02	0.1013D-06
PHI(5,5)	0.1289D-03	0.3042D-03	0.1242D-02	0.5424D-02	-0.3149D-06

Figure II-1. Continued

FREQUENCY =	0.1124D 01	0.1231D 01	0.1313D 01	0.1463D 01	0.1565D 01
GEN COORD	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE
U(1,1)	0.9840D 00	-0.1629D 00	-0.5163D-01	-0.5695D-04	0.4294D-02
U(1,3)	-0.2460D-01	-0.4394D 00	0.8500D 00	0.1460D-03	0.6339D-01
U(1,5)	-0.4759D-02	-0.2299D-01	-0.3679D-01	-0.1241D-03	0.8663D 00
U(3,1)	-0.3024D-02	-0.1708D-01	-0.5932D-02	0.6032D-05	-0.1414D-02
U(3,3)	0.4653D-02	0.2847D-01	0.1349D-01	-0.8355D-05	0.4044D-02
U(3,5)	0.1729D-02	0.8358D-02	0.6580D-02	-0.2806D-05	-0.2862D-02
U(5,1)	-0.6002D-03	-0.2418D-02	-0.1064D-02	0.7728D-06	0.1757D-03
U(5,3)	0.5165D-03	0.4568D-02	0.8725D-03	-0.1251D-05	0.1914D-03
U(5,5)	0.4321D-03	0.1305D-02	0.1272D-02	-0.3760D-06	-0.1739D-02
V(1,1)	0.4997D-01	0.2024D-01	-0.4301D-01	-0.2361D-02	-0.1098D-01
V(1,3)	-0.6724D-02	0.2614D-01	-0.7014D-01	-0.2021D-03	0.4723D 00
V(1,5)	-0.1240D-02	0.3381D-02	-0.4807D-02	-0.4830D-04	-0.2851D-01
V(3,1)	0.1685D 00	0.8812D 00	0.4380D 00	0.1985D-04	0.3720D-01
V(3,3)	0.7301D-02	-0.2865D-01	0.6501D-01	0.2599D-04	-0.1322D 00
V(3,5)	0.1695D-02	-0.6128D-02	0.6465D-02	-0.4824D-05	0.4127D-01
V(5,1)	0.9433D-02	0.3595D-02	-0.2239D-01	0.1941D-05	-0.1906D-01
V(5,3)	0.1776D-02	-0.5070D-02	0.1250D-01	0.3788D-05	-0.1887D-01
V(5,5)	0.6551D-03	-0.9873D-03	0.2485D-02	-0.2125D-05	0.1164D-01
W(1,1)	-0.1814D-02	0.4166D-03	-0.1595D-03	-0.6848D-02	0.4004D-03
W(1,3)	-0.4585D-03	0.8711D-03	-0.1734D-02	0.1246D-01	0.2808D-02
W(1,5)	-0.7759D-04	-0.3109D-03	0.8939D-03	0.5020D-02	-0.4962D-02
W(3,1)	0.4574D-02	0.2983D-02	0.1504D-02	-0.3673D-03	-0.8282D-06
W(3,3)	-0.1470D-02	-0.8120D-02	0.2934D-03	0.5090D-03	-0.7692D-03
W(3,5)	-0.6939D-03	-0.2347D-02	-0.1891D-02	0.1386D-03	0.3114D-02
W(5,1)	0.1591D-02	-0.2892D-03	-0.1623D-03	-0.2756D-03	-0.6079D-04
W(5,3)	-0.7266D-04	-0.5259D-03	0.1360D-02	0.4156D-03	0.5771D-04
W(5,5)	-0.4962D-04	0.5190D-04	-0.8417D-04	0.1271D-03	0.9919D-03
PSI(1,1)	-0.1629D-03	-0.4192D-04	-0.2302D-04	-0.1359D-01	0.4944D-05
PSI(1,3)	0.2240D-04	0.1618D-03	-0.7540D-04	0.2012D-01	0.3041D-04
PSI(1,5)	0.9303D-05	0.2721D-04	0.3058D-04	0.5858D-02	-0.1106D-03
PSI(3,1)	0.3357D-04	0.3772D-04	0.1767D-04	-0.1862D-02	-0.5757D-06
PSI(3,3)	-0.1623D-04	-0.7979D-04	-0.5245D-05	0.3109D-02	-0.1364D-05
PSI(3,5)	-0.6395D-05	-0.2272D-04	-0.1496D-04	0.1085D-02	0.1765D-04
PSI(5,1)	0.3470D-04	0.2124D-05	0.6766D-06	-0.5747D-03	-0.8236D-06
PSI(5,3)	-0.4466D-05	-0.2489D-04	0.2102D-04	0.5873D-03	0.1171D-05
PSI(5,5	-0.2030D-05	-0.4211D-05	-0.4134D-05	0.3677D-03	0.1888D-04
PHI(1,1)	0.1671D-03	0.8944D-04	-0.2438D-03	0.9996D 00	0.1632D-03
PHI(1,3)	-0.1636D-04	0.5163D-04	-0.1421D-03	-0.2927D-03	0.1175D-02
PHI(1,5)	-0.3907D-05	0.2269D-06	0.2431D-05	-0.1334D-03	-0.7342D-04
PHI(3,1)	0.3698D-03	0.2083D-02	0.1133D-02	-0.2334D-02	0.2095D-03
PHI(3,3)	0.7490D-05	-0.7417D-04	0.5659D-04	0.3668D-03	-0.2413D-03
PHI(3,5)	-0.5836D-05	-0.3916D-04	-0.1296D-04	0.2264D-03	0.5169D-04
PHI(5,1)	0.1597D-04	0.8781D-05	-0.3443D-04	-0.5874D-03	-0.3319D-04
PHI(5,3)	0.2415D-05	-0.7599D-05	0.1658D-04	0.1618D-03	-0.2432D-04
PHI(5,5)	0.2285D-06	-0.1707D-05	0.2178D-05	0.1509D-03	0.1242D-04

Figure II-1. Continued

FREQUENCY #	0.1630D 01	0.1719D 01	0.1784D 01	0.1853D 01	0.2001D 01
GEN COORD	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE
U(1,1)	0.5594D-02	0.5447D-04	-0.1007D-01	-0.8357D-02	-0.1609D-04
U(1,3)	0.4781D-01	-0.1171D-03	0.2122D-01	-0.6832D-01	-0.7758D-04
U(1,5)	-0.4627D 00	-0.1548D-03	0.2474D-01	0.1689D 00	0.1327D-03
U(3,1)	-0.1963D-02	0.7313D-04	0.4670D-01	-0.1977D-01	-0.2155D-05
U(3,3)	-0.1552D-01	-0.1069D-03	-0.7364D-01	-0.8828D-01	0.2233D-04
U(3,5)	0.1531D-01	-0.3172D-04	-0.1894D-01	0.8273D-01	-0.1743D-04
U(5,1)	-0.5429D-03	0.4627D-05	-0.1352D-01	-0.8106D-03	-0.1594D-05
U(5,3)	-0.2561D-02	-0.6596D-05	0.2322D-01	-0.7674D-02	0.8369D-06
U(5,5)	0.3924D-02	-0.2484D-05	0.8343D-02	0.7618D-02	-0.2253D-05
V(1,1)	0.3444D-02	-0.4645D-05	-0.4001D-04	0.2691D-03	0.1045D-04
V(1,3)	0.8775D 00	0.1379D-04	-0.3071D-02	-0.3018D-01	-0.2457D-02
V(1,5)	0.1539D-01	0.5616D-05	-0.7952D-03	-0.8159D-02	-0.1431D-03
V(3,1)	-0.9668D-02	-0.2307D-02	0.8510D-02	0.3051D-02	0.3575D-05
V(3,3)	0.1089D 00	-0.5958D-04	0.1972D-01	0.5740D 00	0.1546D-03
V(3,5)	-0.2294D-01	-0.3716D-04	0.3981D-02	0.3876D-02	0.1104D-04
V(5,1)	0.1060D-01	0.1236D-03	0.5546D 00	-0.2636D-01	-0.3035D-05
V(5,3)	0.1035D-01	-0.3196D-05	-0.6644D-02	-0.3221D-01	0.9916D-05
V(5,5)	-0.5683D-02	-0.6610D-05	-0.3071D-02	0.6327D-02	0.2557D-06
W(1,1)	0.7834D-03	-0.5300D-03	-0.7159D-05	-0.1059D-04	0.1595D-04
W(1,3)	0.4558D-02	0.6495D-03	-0.1438D-04	-0.6558D-04	-0.8087D-02
W(1,5)	-0.5859D-02	0.1809D-03	-0.3159D-05	-0.4256D-04	0.1175D-01
W(3,1)	0.7825D-04	-0.3555D-02	-0.7372D-04	0.7033D-03	-0.2648D-02
W(3,3)	0.7403D-03	0.7073D-02	0.2359D-03	0.4390D-02	-0.2198D-02
W(3,5)	-0.1887D-02	0.3045D-02	-0.3396D-04	-0.5518D-02	0.6836D-03
W(5,1)	0.2995D-04	-0.1033D-02	0.2047D-02	-0.1185D-03	-0.1120D-02
W(5,3)	-0.2365D-05	0.3085D-03	-0.3658D-02	-0.2938D-03	-0.1142D-02
W(5,5)	-0.4526D-03	0.1456D-03	-0.1359D-02	0.5534D-03	0.5824D-03
PSI(1,1)	-0.9426D-06	0.4156D-01	-0.2051D-04	-0.1403D-05	-0.1428D 00
PSI(1,3)	0.1722D-04	-0.5479D-01	0.2299D-04	-0.2540D-04	-0.1390D 00
PSI(1,5)	0.3158D-04	-0.1386D-01	0.4848D-05	-0.4109D-05	0.6737D-01
PSI(3,1)	0.2066D-05	-0.1404D-01	-0.2093D-04	0.3671D-05	-0.8477D-03
PSI(3,3)	0.1036D-04	0.2337D-01	0.3342D-04	0.2373D-04	-0.9439D-02
PSI(3,5)	-0.1776D-04	0.7913D-02	0.9558D-05	-0.1653D-04	0.9614D-02
PSI(5,1)	0.7430D-06	-0.2394D-02	0.1064D-04	-0.8426D-06	-0.2599D-03
PSI(5,3)	0.2376D-05	0.4168D-02	-0.1752D-04	-0.2453D-05	-0.1287D-02
PSI(5,5)	-0.1107D-04	0.1525D-02	-0.5766D-05	0.4528D-05	0.1538D-02
PHI(1,1)	0.9232D-04	0.3947D-02	-0.2396D-05	-0.1535D-05	0.8088D-03
PHI(1,3)	0.2382D-02	0.1614D-02	-0.1347D-04	-0.1612D-03	0.9769D 00
PHI(1,5)	-0.5412D-04	0.4839D-03	-0.1097D-05	-0.7276D-05	-0.6583D-03
PHI(3,1)	-0.1009D-03	0.9971D 00	-0.1030D-03	0.7686D-04	-0.2347D-02
PHI(3,3)	0.2087D-03	-0.2936D-02	0.4303D-04	0.2477D-02	-0.3280D-01
PHI(3,5)	-0.3116D-04	-0.1627D-02	0.2500D-05	-0.5746D-04	0.8515D-03
PHI(5,1)	0.1877D-04	-0.8469D-03	0.2267D-02	-0.7080D-04	0.2342D-02
PHI(5,3)	0.1492D-04	0.1389D-03	-0.2273D-04	-0.4235D-04	-0.5578D-02
PHI(5,5)	-0.6307D-05	0.2424D-03	-0.2087D-04	0.7006D-05	0.5055D-03

Figure II-1. Continued

FREQUENCY =	0.2021D 01	0.2114D 01	0.2147D 01	0.2157D 01	0.2209D 01
GEN COORD	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE
L(1,1)	0.4327D-04	0.1775D-05	0.1047D-04	-0.3956D-07	-0.1953D-02
L(1,3)	-0.4588D-05	0.6234D-04	-0.7429D-05	0.5602D-05	-0.1410D-01
L(1,5)	0.1489D-04	-0.3673D-04	-0.2512D-04	-0.3253D-04	0.2442D-01
L(3,1)	0.4724D-04	0.1551D-03	-0.4929D-03	0.1699D-02	0.9312D 00
L(3,3)	0.2910D-04	0.1282D-03	0.5237D-03	0.1752D-02	0.3306D 00
L(3,5)	-0.4671D-05	-0.1293D-03	0.9546D-05	-0.9322D-03	-0.1570D 00
L(5,1)	0.1846D-04	-0.6704D-05	0.3062D-04	-0.2262D-04	-0.5347D-02
L(5,3)	-0.4873D-06	0.2966D-04	-0.5392D-04	-0.1112D-03	-0.3230D-01
L(5,5)	-0.1451D-05	0.1567D-05	-0.1441D-04	0.1308D-03	0.3752D-01
V(1,1)	-0.2911D-04	0.4028D-04	0.3625D-05	-0.1799D-04	0.2003D-03
V(1,3)	-0.3562D-03	-0.3313D-03	-0.5231D-04	0.4848D-04	0.5292D-02
V(1,5)	-0.1761D-04	-0.5667D-05	-0.6014D-05	-0.3254D-06	-0.1386D-02
V(3,1)	0.9598D-04	-0.1051D-03	-0.5878D-04	0.3323D-04	0.1089D-02
V(3,3)	-0.4693D-04	-0.7886D-03	-0.3024D-03	-0.1817D-02	0.7352D-01
V(3,5)	-0.7475D-05	-0.4354D-04	-0.3344D-04	-0.1034D-04	0.7444D-02
V(5,1)	-0.7987D-04	0.5051D-03	-0.2118D-02	0.2291D-03	-0.9188D-03
V(5,3)	0.9119D-06	0.9444D-04	0.1487D-03	0.3070D-02	0.7583D 00
V(5,5)	-0.2334D-05	-0.1348D-04	-0.6283D-05	-0.4659D-04	-0.7659D-02
W(1,1)	-0.1049D-01	-0.6546D-05	0.2804D-03	0.1636D-03	-0.1096D-03
W(1,3)	-0.1171D-02	-0.5896D-02	-0.1743D-02	0.3237D-02	-0.5459D-04
W(1,5)	0.1769D-02	0.1987D-02	0.6842D-03	0.1969D-02	-0.2188D-04
W(3,1)	0.1861D-01	-0.5654D-03	-0.1401D-02	-0.1114D-02	-0.6655D-03
W(3,3)	-0.5572D-03	0.1315D-01	0.2846D-02	-0.1178D-01	-0.1761D-03
W(3,5)	0.2489D-03	0.2524D-02	0.9311D-03	0.4238D-02	-0.5496D-04
W(5,1)	0.7373D-02	0.2957D-03	-0.2310D-02	-0.1404D-03	0.1339D-02
W(5,3)	0.3912D-05	0.4825D-02	0.4852D-02	-0.3568D-02	0.3054D-02
W(5,5)	0.1848D-03	-0.1089D-03	0.1560D-02	-0.7641D-03	-0.3493D-02
RSI(1,1)	0.9875D 00	-0.9234D-02	-0.4028D-01	-0.1838D-01	0.4015D-04
RSI(1,3)	-0.8596D-02	0.9065D 00	0.1793D 00	-0.3465D 00	0.1130D-02
RSI(1,5)	0.8598D-02	-0.4022D-01	-0.4502D-01	-0.2713D 00	0.1162D-02
RSI(3,1)	0.1494D-02	-0.9486D-02	0.2540D-01	-0.1026D-01	-0.3154D-04
RSI(3,3)	-0.4377D-02	-0.3753D-02	-0.4938D-01	-0.4341D-01	0.1401D-03
RSI(3,5)	0.9569D-03	0.1900D-01	-0.6405D-02	0.5061D-01	-0.1989D-03
RSI(5,1)	-0.1742D-03	0.1687D-02	-0.1144D-01	0.5232D-04	0.1435D-04
RSI(5,3)	0.4026D-04	-0.7070D-02	0.1839D-01	-0.7713D-02	0.3811D-04
RSI(5,5)	0.6706D-03	0.1889D-02	0.8412D-02	0.6825D-02	-0.2732D-04
RHI(1,1)	0.1346D-01	-0.1815D-01	-0.3142D-02	0.7929D-02	-0.2783D-04
RHI(1,3)	0.1431D 00	0.1418D 00	0.2501D-01	-0.3875D-02	-0.3255D-04
RHI(1,5)	-0.2264D-02	-0.7452D-02	0.3128D-03	0.1560D-01	-0.6708D-04
RHI(3,1)	-0.4152D-01	0.4999D-01	0.1354D-01	-0.1900D-01	0.6461D-04
RHI(3,3)	0.2251D-01	0.3214D 00	0.1422D 00	0.8906D 00	-0.3422D-02
RHI(3,5)	0.3373D-02	0.8590D-02	0.8027D-03	-0.2523D-01	0.1087D-03
RHI(5,1)	0.3644D-01	-0.2213D 00	0.9692D 00	-0.8095D-01	0.3440D-03
RHI(5,3)	0.2263D-02	0.2059D-01	0.6958D-03	-0.1202D-01	0.1686D-02
RHI(5,5)	0.8927D-03	0.3852D-02	-0.2271D-02	-0.3212D-02	-0.2807D-04

Figure II-1. Continued

FREQUENCY =	0.2224D 01	0.2320D 01	0.2334D 01	0.2396D 01	0.2471D 01
GEN COORD	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE
L(1,1)	0.1138D-02	0.3974D-05	0.5520D-03	0.1433D-02	0.1024D-02
L(1,3)	0.7942D-02	0.2370D-04	0.3420D-02	0.6263D-02	0.4036D-02
L(1,5)	-0.1284D-01	0.2659D-05	-0.1655D-02	0.3119D-01	0.2601D-01
L(3,1)	0.8451D 00	-0.2241D-04	0.4820D-02	0.6279D-02	-0.1522D-01
L(3,3)	-0.2142D 00	0.1519D-02	0.8965D 00	0.6806D-01	-0.1440D 00
L(3,5)	0.1162D 00	0.2220D-03	0.2132D 00	-0.7599D-01	0.8191D 00
L(5,1)	0.4208D-02	0.3492D-05	0.1307D-02	0.3469D-05	-0.2417D-02
L(5,3)	0.1835D-01	0.3095D-04	0.1686D-01	0.1143D-02	-0.1238D-01
L(5,5)	-0.2411D-01	-0.1559D-04	-0.1868D-01	-0.6685D-02	0.1198D-01
V(1,1)	0.2785D-02	0.1158D-05	-0.3928D-02	-0.2967D-03	-0.7738D-03
V(1,3)	-0.1591D-02	0.1438D-03	0.6234D-02	0.7382D-03	-0.9945D-02
V(1,5)	-0.4151D-02	-0.8233D-04	-0.6080D-01	0.9924D 00	0.9979D-01
V(3,1)	0.2257D-01	-0.7168D-04	-0.3029D-01	-0.6132D-03	-0.4746D-02
V(3,3)	-0.2625D-01	-0.5017D-03	0.5407D-01	0.1453D-01	-0.7711D-01
V(3,5)	-0.1976D-01	-0.2464D-03	-0.1044D 00	0.2800D-01	-0.4555D 00
V(5,1)	-0.5774D-01	0.8832D-04	0.6737D-01	0.2465D-02	0.1088D-01
V(5,3)	-0.4692D 00	-0.5504D-03	-0.3520D 00	-0.5025D-01	0.2575D 00
V(5,5)	0.1499D-01	0.5296D-04	0.9758D-01	-0.6311D-02	0.1306D 00
W(1,1)	-0.1773D-03	0.1870D-03	-0.1998D-04	0.2577D-03	0.2300D-04
W(1,3)	0.2341D-04	0.1013D-02	-0.2179D-03	0.9975D-03	0.9639D-04
W(1,5)	-0.2937D-04	-0.7514D-02	-0.3137D-03	0.4145D-02	0.2699D-03
W(3,1)	-0.1144D-02	-0.3950D-03	-0.4319D-04	0.6690D-05	-0.1251D-03
W(3,3)	0.1333D-03	-0.2294D-02	-0.1011D-02	-0.6079D-05	-0.4611D-03
W(3,5)	-0.1681D-03	0.1504D-01	-0.5199D-03	0.1881D-03	-0.2431D-02
W(5,1)	0.1206D-02	-0.1077D-03	-0.5749D-04	-0.9749D-05	0.1195D-03
W(5,3)	-0.1729D-02	-0.3565D-03	0.3633D-03	-0.3595D-04	0.9613D-03
W(5,5)	0.2270D-02	0.5177D-02	0.1862D-02	0.5084D-04	0.8600D-03
PSI(1,1)	-0.5711D-04	-0.5823D-02	0.1858D-04	0.4926D-05	-0.8628D-05
PSI(1,3)	-0.7245D-04	-0.4299D-01	0.9360D-04	0.2861D-04	-0.7553D-04
PSI(1,5)	-0.1707D-03	0.9513D 00	-0.1539D-02	0.1459D-03	0.2510D-03
PSI(3,1)	-0.1098D-03	-0.4112D-02	-0.1403D-04	-0.5108D-05	0.4001D-04
PSI(3,3)	-0.3877D-04	-0.2267D-01	-0.7949D-04	-0.2362D-04	0.2127D-03
PSI(3,5)	0.1579D-04	0.1883D-01	-0.3229D-04	0.1059D-04	-0.2742D-03
PSI(5,1)	0.1731D-04	-0.2851D-03	0.8413D-05	0.1305D-05	-0.1151D-04
PSI(5,3)	0.5260D-05	-0.1014D-04	0.3489D-04	0.6073D-05	-0.7308D-04
PSI(5,5)	0.6468D-05	-0.1198D-02	-0.1696D-04	-0.1027D-04	0.1071D-03
PHI(1,1)	-0.4116D-06	-0.4820D-02	0.8846D-05	0.1828D-04	0.1277D-05
PHI(1,3)	0.1184D-04	-0.6401D-01	0.7921D-04	0.9070D-04	0.1236D-04
PHI(1,5)	0.2272D-05	-0.5696D-01	-0.3040D-04	0.2386D-02	0.2992D-03
PHI(3,1)	-0.3257D-04	0.1248D-01	0.9313D-05	0.4571D-05	-0.9098D-05
PHI(3,3)	0.4482D-03	0.2740D 00	-0.7188D-03	-0.2438D-04	0.2146D-03
PHI(3,5)	-0.3721D-04	0.6831D-01	-0.2658D-03	0.5741D-04	-0.7967D-03
PHI(5,1)	0.4283D-03	0.1230D-01	-0.2598D-03	-0.7281D-05	0.5537D-05
PHI(5,3)	-0.1069D-02	-0.6477D-01	-0.1056D-02	-0.2424D-03	0.2438D-02
PHI(5,5)	0.3160D-04	0.1810D-01	0.4099D-04	-0.5915D-07	0.1058D-03

Figure II-1. Continued

FREQUENCY =	0.2533D 01	0.2591D 01	0.2676D 01	0.2782D 01	0.2812D 01
GEN COORD	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE
U(1,1)	-0.5418D-06	-0.2026D-02	-0.1933D-05	-0.4618D-05	0.3746D-05
U(1,3)	0.6649D-05	-0.8608D-02	-0.8266D-05	-0.2649D-05	0.1859D-04
U(1,5)	-0.1154D-03	-0.4205D-01	-0.4336D-04	-0.1222D-04	0.8738D-04
U(3,1)	0.8615D-04	0.4474D-02	-0.3300D-05	0.8884D-04	0.2744D-05
U(3,3)	0.6853D-03	0.2102D-01	-0.1312D-04	0.4248D-05	-0.9340D-04
U(3,5)	-0.1058D-02	0.4483D 00	-0.1265D-03	0.4509D-04	-0.2383D-03
U(5,1)	0.2281D-04	-0.2987D-02	0.1542D-05	0.4493D-04	0.1573D-04
U(5,3)	0.1353D-03	-0.1112D-01	0.5563D-05	-0.5349D-05	0.2567D-04
U(5,5)	-0.1651D-03	-0.2104D-01	0.1778D-04	-0.1103D-04	0.1086D-03
V(1,1)	-0.1981D-06	-0.3895D-03	0.5350D-05	-0.2206D-05	-0.2626D-05
V(1,3)	0.1973D-04	-0.2835D-02	0.3372D-04	-0.9028D-06	-0.3780D-05
V(1,5)	-0.6493D-04	0.1432D-01	-0.2274D-02	-0.1853D-04	0.9543D-04
V(3,1)	-0.2733D-05	-0.4817D-02	-0.8050D-06	-0.1951D-04	-0.3453D-05
V(3,3)	0.5702D-04	-0.3068D-01	-0.2785D-05	-0.1772D-04	0.3426D-04
V(3,5)	0.1662D-02	0.8811D 00	-0.1337D-03	0.3032D-03	-0.2294D-02
V(5,1)	0.1692D-05	0.9476D-02	-0.2119D-05	0.4084D-04	0.2274D-04
V(5,3)	-0.2687D-02	0.7778D-01	-0.1861D-04	0.4554D-04	-0.6609D-05
V(5,5)	-0.2227D-03	0.1117D 00	-0.5482D-04	0.9482D-04	-0.6578D-03
W(1,1)	-0.3025D-04	0.2239D-05	-0.5425D-03	-0.2177D-02	-0.2285D-03
W(1,3)	-0.6811D-04	0.8091D-05	-0.2144D-02	-0.6928D-04	0.6704D-03
W(1,5)	-0.1350D-03	0.8535D-05	-0.9001D-02	-0.1961D-03	0.1138D-02
W(3,1)	-0.4727D-03	0.1789D-03	0.9347D-04	-0.1396D-01	-0.2142D-02
W(3,3)	-0.2406D-02	0.6883D-03	0.2260D-03	-0.4087D-04	0.1685D-02
W(3,5)	0.2419D-02	0.2778D-02	0.8554D-03	0.5329D-03	-0.4559D-02
W(5,1)	-0.2515D-03	0.7084D-04	-0.9989D-04	0.1829D-01	0.2212D-02
W(5,3)	-0.2536D-02	0.3038D-03	-0.1877D-03	0.3481D-03	-0.4509D-02
W(5,5)	0.4734D-02	0.7731D-03	-0.1804D-03	0.3556D-03	-0.4030D-02
PSI(1,1)	-0.3508D-02	-0.3036D-05	0.2032D-02	0.2223D-03	-0.2882D-02
PSI(1,3)	-0.2712D-01	-0.7354D-05	0.9176D-02	0.1663D-02	-0.1239D-01
PSI(1,5)	0.5958D-01	-0.1772D-03	0.5697D-01	0.8656D-02	-0.6859D-01
PSI(3,1)	0.2533D-01	-0.6097D-04	-0.5935D-02	0.5917D 00	0.1186D 00
PSI(3,3)	0.1205D 00	-0.2111D-03	-0.1290D-01	0.1958D-01	-0.2572D 00
PSI(3,5)	-0.1105D 00	-0.6637D-04	-0.2622D-01	0.3743D-01	-0.2426D 00
PSI(5,1)	-0.5763D-02	0.5069D-05	-0.2756D-03	0.5803D-03	-0.2592D-03
PSI(5,3)	-0.3440D-01	0.2898D-04	-0.1040D-02	0.2918D-03	-0.2837D-02
PSI(5,5)	0.4099D-01	-0.4004D-04	-0.3661D-02	-0.8073D-03	-0.1070D-01
PHI(1,1)	-0.2158D-03	0.1569D-05	-0.2353D-03	0.1651D-02	0.1692D-02
PHI(1,3)	-0.5757D-03	0.8050D-05	-0.1297D-02	0.7418D-03	0.1898D-02
PHI(1,5)	-0.4235D-02	0.1668D-03	0.9975D 00	0.9289D-02	-0.2763D-01
PHI(3,1)	-0.2027D-02	0.2107D-04	0.1219D-02	0.1316D-01	0.9639D-02
PHI(3,3)	0.3370D-01	0.8819D-04	0.4568D-02	0.7946D-02	0.1395D-02
PHI(3,5)	0.1825D-01	0.2332D-02	0.2436D-01	-0.1109D 00	0.9223D 00
PHI(5,1)	0.6975D-02	-0.2400D-04	-0.6504D-03	-0.2580D-01	-0.1595D-01
PHI(5,3)	0.9817D 00	-0.8084D-03	-0.6344D-03	-0.2198D-01	-0.1249D-01
PHI(5,5)	-0.1180D-01	0.1237D-02	-0.1756D-02	0.2017D-01	-0.6172D-01

Figure II-1. Continued

FREQUENCY =	0.28670D 01	0.28700D 01	0.29920D 01	0.31270D 01	0.33180D 01
GEN COORD	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE
L(1,1)	-0.29720D-05	-0.63810D-03	0.15070D-05	0.90590D-06	-0.51980D-04
L(1,3)	-0.15620D-04	-0.26110D-02	0.63090D-05	0.36530D-05	-0.74210D-05
L(1,5)	-0.51260D-04	-0.11170D-01	0.24730D-04	0.16850D-04	-0.11150D-04
L(3,1)	-0.41060D-04	-0.65800D-02	0.81290D-05	0.75720D-05	-0.22960D-03
L(3,3)	-0.12790D-03	-0.29350D-01	0.35440D-04	0.31680D-04	-0.53680D-04
L(3,5)	-0.10900D-02	-0.17650D 00	0.20310D-03	0.16780D-03	-0.11590D-03
L(5,1)	-0.55370D-04	-0.94940D-02	0.22340D-04	0.36500D-04	0.99980D 00
L(5,3)	-0.15600D-03	-0.31890D-01	0.76620D-04	0.10380D-03	-0.89440D-03
L(5,5)	-0.58470D-03	-0.10430D 00	0.22540D-03	0.29010D-03	-0.49530D-03
V(1,1)	0.60480D-05	0.39090D-03	0.31240D-06	-0.13160D-05	0.65110D-03
V(1,3)	-0.99810D-06	0.14030D-02	0.65260D-05	-0.52080D-05	0.31780D-03
V(1,5)	-0.40380D-04	-0.49070D-02	-0.23500D-04	0.17410D-04	0.32120D-03
V(3,1)	0.44320D-04	0.21130D-02	0.58320D-05	-0.76490D-05	0.27540D-02
V(3,3)	-0.15210D-04	0.82090D-02	0.64590D-04	-0.33690D-04	0.13710D-02
V(3,5)	-0.79200D-03	-0.35930D-01	-0.40010D-03	0.14150D-03	0.14780D-02
V(5,1)	-0.63630D-04	-0.43040D-03	-0.12940D-04	0.82850D-05	0.13650D-01
V(5,3)	0.17160D-03	-0.54630D-02	-0.15580D-03	0.55280D-04	0.71620D-02
V(5,5)	0.57460D-02	0.97690D 00	-0.15400D-02	-0.19030D-02	0.96940D-02
W(1,1)	-0.14400D-04	0.17960D-05	0.26060D-04	0.25350D-04	-0.60260D-04
W(1,3)	-0.19990D-02	0.18310D-04	0.16680D-03	0.11430D-03	0.40010D-07
W(1,5)	0.12610D-03	0.15910D-04	-0.16240D-02	0.98830D-03	-0.74800D-07
W(3,1)	-0.23100D-03	0.69190D-05	0.74550D-04	0.13180D-03	-0.23480D-03
W(3,3)	-0.13490D-01	0.10210D-03	0.66440D-03	0.61210D-03	0.25090D-06
W(3,5)	-0.17020D-02	0.45340D-04	-0.12330D-01	0.58520D-02	-0.11570D-06
W(5,1)	0.13040D-03	0.18190D-03	-0.38310D-03	-0.36700D-03	-0.96550D-03
W(5,3)	0.17010D-01	0.59490D-03	-0.20350D-02	-0.15980D-02	0.27780D-05
W(5,5)	-0.11280D-02	0.28440D-02	0.11460D-01	-0.11180D-01	0.37470D-05
RSI(1,1)	-0.76540D-03	0.89330D-06	-0.98330D-03	-0.62800D-03	-0.63660D-05
RSI(1,3)	-0.36660D-02	0.58720D-05	-0.42060D-02	-0.26020D-02	0.87940D-07
RSI(1,5)	-0.19190D-01	0.33580D-04	-0.21870D-01	-0.11950D-01	0.12850D-06
RSI(3,1)	0.78490D-02	-0.91090D-04	-0.10240D-01	-0.91430D-02	-0.33080D-04
RSI(3,3)	0.95190D 00	-0.59590D-02	-0.66280D-01	-0.44890D-01	0.13700D-05
RSI(3,5)	-0.10890D-01	0.32190D-03	0.84760D 00	-0.45210D 00	0.30990D-05
RSI(5,1)	-0.43600D-03	-0.56870D-05	-0.39300D-02	-0.54930D-02	-0.62720D-04
RSI(5,3)	0.43390D-02	-0.57870D-04	-0.14950D-01	-0.19250D-01	0.80090D-06
RSI(5,5)	-0.13160D-01	-0.47270D-05	-0.30280D-01	-0.72910D-01	0.17290D-05
PHI(1,1)	-0.28280D-02	0.17620D-04	-0.62820D-03	0.65250D-03	-0.23980D-06
PHI(1,3)	0.66020D-02	-0.42360D-04	-0.59520D-02	0.28850D-02	-0.15080D-06
PHI(1,5)	0.65660D-02	-0.28850D-04	0.17850D-01	-0.88590D-02	-0.12350D-06
PHI(3,1)	-0.21840D-01	0.13210D-03	-0.55060D-02	0.41070D-02	-0.11210D-05
PHI(3,3)	0.49680D-01	-0.33180D-03	-0.49450D-01	0.15560D-01	-0.80110D-06
PHI(3,5)	0.26350D 00	-0.82980D-03	0.23580D 00	-0.73900D-01	-0.10080D-05
PHI(5,1)	0.40890D-01	-0.23320D-03	0.10880D-01	-0.90270D-02	-0.45220D-05
PHI(5,3)	-0.12270D 00	0.89050D-03	0.10870D 00	-0.30790D-01	-0.39800D-05
PHI(5,5)	0.58730D-01	0.20480D-02	0.45220D 00	0.88350D 00	-0.20320D-04

Figure II-1. Continued

FREQUENCY =	0.2285D 01	0.3514D 01	0.3718D 01	0.3777D 01	0.3852D 01
GEN COORD	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE
L(1,1)	-0.4429D-05	0.4293D-05	-0.2820D-05	0.2311D-08	-0.2597D-08
L(1,3)	-0.4985D-03	0.2208D-04	0.3307D-08	-0.2531D-05	-0.1296D-07
L(1,5)	-0.3299D-05	-0.1577D-02	-0.9178D-08	-0.3020D-08	-0.1751D-05
L(3,1)	-0.3254D-04	0.8712D-06	-0.9865D-05	-0.1257D-06	0.3001D-06
L(3,3)	-0.2229D-02	0.3203D-04	-0.2811D-06	-0.8590D-05	0.1021D-06
L(3,5)	-0.1584D-03	-0.7234D-02	0.2994D-07	0.1482D-07	-0.5758D-05
L(5,1)	0.5769D-03	-0.7594D-04	0.5023D-04	0.1588D-07	0.1701D-06
L(5,3)	0.9982D 00	-0.7269D-03	0.3764D-06	0.4431D-04	0.2210D-06
L(5,5)	-0.8468D-03	0.9930D 00	0.1481D-06	0.2226D-05	0.3081D-04
V(1,1)	-0.1113D-02	-0.4002D-03	-0.4752D-06	0.8030D-06	0.2786D-06
V(1,3)	0.1932D-02	-0.2232D-02	-0.2375D-06	-0.1412D-05	0.1557D-05
V(1,5)	0.1156D-02	0.4036D-02	-0.5645D-06	-0.8473D-06	-0.2528D-05
V(3,1)	-0.4691D-02	-0.1678D-02	-0.2010D-05	0.3402D-05	0.1206D-05
V(3,3)	0.8293D-02	-0.9502D-02	-0.1040D-05	-0.6062D-05	0.6648D-05
V(3,5)	0.5256D-02	0.1796D-01	-0.1078D-05	-0.3833D-05	-0.1302D-04
V(5,1)	-0.2319D-01	-0.8284D-02	-0.5833D-05	0.1654D-04	0.9828D-05
V(5,3)	0.4298D-01	-0.4878D-01	-0.5169D-05	-0.3087D-04	0.3509D-04
V(5,5)	0.3276D-01	0.1052D 00	-0.6743D-05	-0.2287D-04	-0.7574D-04
W(1,1)	-0.5313D-07	-0.1217D-06	-0.7857D-03	0.7409D-07	0.4348D-06
W(1,3)	-0.5387D-04	-0.4392D-06	0.2104D-06	-0.7648D-03	0.2071D-05
W(1,5)	-0.4242D-06	-0.4146D-04	0.1547D-06	-0.8652D-07	-0.7257D-03
W(3,1)	-0.1326D-06	-0.3923D-06	-0.3062D-02	0.2381D-06	0.1715D-05
W(3,3)	-0.2084D-03	-0.1509D-05	0.7535D-06	-0.2984D-02	0.8226D-05
W(3,5)	-0.1208D-05	-0.1562D-03	0.6556D-06	-0.3288D-06	-0.2840D-02
W(5,1)	0.7099D-06	-0.2799D-06	-0.1260D-01	-0.4231D-06	0.7006D-05
W(5,3)	-0.8426D-03	-0.1839D-05	0.1032D-05	-0.1236D-01	0.3547D-04
W(5,5)	0.4667D-05	-0.5823D-03	-0.1557D-06	-0.4686D-05	-0.1157D-01
RSI(1,1)	0.1943D-07	0.2671D-06	-0.1024D-03	0.6396D-06	0.9434D-05
RSI(1,3)	-0.5158D-05	0.1362D-06	-0.4787D-06	-0.4666D-03	0.3937D-04
RSI(1,5)	0.5244D-06	-0.2217D-05	0.5587D-05	0.3424D-04	-0.1307D-02
RSI(3,1)	0.6130D-06	0.2592D-06	-0.4202D-03	-0.4434D-05	0.2925D-04
RSI(3,3)	-0.2597D-04	0.1124D-05	-0.1169D-04	-0.2015D-02	0.1318D-03
RSI(3,5)	0.5188D-05	-0.6850D-05	0.2881D-05	0.9511D-04	-0.9793D-02
RSI(5,1)	0.1911D-06	0.8065D-06	0.9998D 00	0.1870D-04	-0.2905D-03
RSI(5,3)	-0.5073D-04	0.7076D-06	-0.1477D-03	0.9988D 00	-0.1757D-02
RSI(5,5)	0.4183D-05	-0.1836D-04	0.9262D-04	0.1234D-02	0.9958D 00
PHI(1,1)	0.3525D-06	0.9542D-07	0.5727D-03	-0.5885D-03	-0.3558D-03
PHI(1,3)	-0.8375D-06	0.7307D-06	0.2724D-03	0.1668D-02	-0.1572D-02
PHI(1,5)	-0.5823D-07	-0.2130D-05	0.2583D-03	0.5421D-03	0.3363D-02
PHI(3,1)	0.1628D-05	0.4150D-06	0.2397D-02	-0.4125D-02	-0.1456D-02
PHI(3,3)	-0.4049D-05	0.3331D-05	0.1157D-02	0.7059D-02	-0.8255D-02
PHI(3,5)	-0.4115D-05	-0.1146D-04	0.1146D-02	0.4146D-02	0.1461D-01
PHI(5,1)	0.8059D-05	0.2665D-05	0.1158D-01	-0.1590D-01	-0.7210D-02
PHI(5,3)	-0.2206D-04	0.2186D-04	0.5809D-02	0.3525D-01	-0.4114D-01
PHI(5,5)	-0.5109D-04	-0.1098D-03	0.6541D-02	0.2312D-01	0.7901D-01

Figure II-1. Continued

DIMENSIONLESS FREQUENCIES AND NORMALIZED EIGENVECTORS
FOR
A CYLINDRICALLY CURVED SANDWICH PANEL
WITH
CLAMPED EDGES

****

NONDIMENSIONAL INPUT PARAMETERS

SUBTENDED ANGLE = 0.2744 ASPECT RATIO = 1.3939
LENGTH/SKIN THICKNESS = 1031.25 CORE/SKIN THICKNESS RATIO = 23.250
CORE/SKIN DENSITY RATIO = .0124740 POISSONS RATIO = .322
RATIO OF CORE TRANSVERSE SHEAR MODULI(GXZ/GYZ) = 1.98895
CORE SHEAR MODULUS/SKIN YOUNGS MODULUS = .00050071

NUMBER OF SERIES TERMS ALONG STRAIGHT EDGE = 3, ALONG CURVED EDGE = 3

****

COMPUTED FREQUENCIES AND MODE SHAPES

FREQUENCY =	0.5339D-01	0.1026D 00	0.1502D 00	0.1632D 00	0.1761D 00
GEN CCORD	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE
U(1,1)	0.2693D-02	0.2713D-02	-0.3947D-02	0.1267D-02	-0.7498D-03
U(1,3)	0.1991D-02	-0.2425D-02	-0.8448D-03	0.1060D-02	0.9990D-04
U(1,5)	0.9552D-03	0.1184D-03	-0.3517D-03	-0.2302D-02	-0.3856D-03
U(3,1)	-0.2090D-03	-0.7620D-04	0.3306D-02	0.2275D-04	0.7029D-03
U(3,3)	-0.4317D-04	0.1972D-04	0.9011D-03	-0.2932D-04	-0.2402D-03
U(3,5)	0.1137D-04	-0.2920D-04	0.5610D-03	0.3192D-05	0.5917D-03
U(5,1)	-0.5049D-04	-0.1235D-04	-0.8212D-07	-0.6353D-05	0.2296D-04
U(5,3)	-0.1486D-04	0.3166D-05	0.1282D-04	-0.1630D-04	-0.5191D-04
U(5,5)	-0.2108D-05	-0.1109D-04	0.9149D-05	0.2134D-04	-0.1504D-05
V(1,1)	0.1888D-01	-0.1927D-01	0.1193D-02	-0.1110D-01	-0.1987D-03
V(1,3)	-0.8346D-03	0.1738D-01	0.2207D-03	-0.7254D-02	-0.4230D-03
V(1,5)	-0.2660D-03	0.6372D-03	-0.1238D-03	0.1344D-01	-0.1291D-04
V(3,1)	0.1783D-02	-0.1533D-02	0.7547D-02	-0.8557D-03	-0.7283D-02
V(3,3)	0.9897D-04	0.9600D-03	0.4032D-03	-0.3588D-03	0.1224D-01
V(3,5)	0.3864D-04	0.7761D-04	-0.4033D-04	0.8221D-03	0.3952D-03
V(5,1)	0.6862D-03	-0.5605D-03	0.9840D-03	-0.3132D-03	-0.9434D-03
V(5,3)	0.7666D-04	0.3201D-03	0.6623D-04	-0.1108D-03	0.1405D-02
V(5,5)	0.4201D-04	0.3057D-04	0.1666D-04	0.2071D-03	0.7310D-04
W(1,1)	$\boxed{0.9965D\ 00}$	-0.5947D-01	-0.4444D-01	-0.9396D-02	0.8445D-02
W(1,3)	0.5849D-01	$\boxed{0.9858D\ 00}$	0.2604D-02	-0.1418D 00	-0.5670D-01
W(1,5)	0.1778D-01	0.1408D 00	-0.1225D-01	$\boxed{0.9864D\ 00}$	-0.1139D-01
W(3,1)	0.4383D-01	-0.9659D-02	$\boxed{0.9871D\ 00}$	0.1020D-01	-0.1091D 00
W(3,3)	0.1473D-03	0.5554D-01	0.1061D 00	-0.5616D-02	$\boxed{0.9775D\ 00}$
W(3,5)	-0.6236D-03	0.8265D-02	0.3416D-01	0.7410D-01	0.1325D 00
W(5,1)	0.1464D-01	-0.2366D-02	0.9625D-01	0.2426D-03	-0.1141D-01
W(5,3)	0.1759D-03	0.1775D-01	0.1011D-01	-0.2696D-02	0.9787D-01
W(5,5)	-0.1939D-03	0.2650D-02	0.2904D-02	0.2285D-01	0.1326D-01
PSI(1,1)	-0.2418D-01	0.8014D-03	0.1539D-01	0.1255D-04	-0.1468D-02
PSI(1,3)	-0.1793D-02	-0.2170D-01	0.1549D-02	0.2736D-02	0.1344D-01
PSI(1,5)	-0.7440D-03	-0.3018D-02	0.7865D-03	-0.1797D-01	0.1893D-02
PSI(3,1)	-0.2396D-03	0.2483D-03	-0.3751D-01	-0.4385D-03	0.3720D-02
PSI(3,3)	0.4248D-04	-0.8054D-03	-0.4183D-02	0.4062D-04	-0.3516D-01
PSI(3,5)	0.8447D-05	-0.1089D-03	-0.1468D-02	-0.1473D-02	-0.4725D-02
PSI(5,1)	-0.1276D-04	0.2418D-04	-0.9690D-03	0.9874D-06	0.1179D-03
PSI(5,3)	0.3012D-05	-0.1574D-03	-0.1207D-03	0.4573D-04	-0.1166D-02
PSI(5,5)	-0.4959D-05	-0.1822D-04	-0.5036D-04	-0.3386D-03	-0.1455D-03
PHI(1,1)	-0.1225D-01	0.6870D-02	-0.1273D-02	0.6053D-02	0.5222D-03
PHI(1,3)	-0.1826D-02	0.6090D-03	-0.1956D-03	0.1015D-02	0.5257D-04
PHI(1,5)	-0.6227D-03	0.3046D-03	-0.6416D-04	0.2046D-03	0.2430D-04
PHI(3,1)	-0.1752D-02	0.9686D-03	-0.4526D-02	0.7536D-03	0.3014D-02
PHI(3,3)	-0.2612D-03	0.8541D-04	-0.6844D-03	0.1267D-03	0.2327D-03
PHI(3,5)	-0.8908D-04	0.4289D-04	-0.2314D-03	0.2435D-04	0.1304D-03
PHI(5,1)	-0.8635D-03	0.4612D-03	-0.1290D-02	0.3564D-03	0.7864D-03
PHI(5,3)	-0.1290D-03	0.3144D-04	-0.1962D-03	0.6192D-04	0.4784D-04
PHI(5,5)	-0.4398D-04	0.1945D-04	-0.6611D-04	0.9842D-05	0.3267D-04

Figure II-2. Computer Output for Beam Function-Clamped Panel

FREQUENCY =	0.2191D 00	0.2744D 00	0.2896D 00	0.3177D 00	0.8344D 00
GEN COORD	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE
U(1,1)	-0.5649D-03	-0.2834D-02	-0.5107D-05	-0.1332D-03	-0.2496D 00
U(1,3)	-0.9925D-03	-0.3811D-03	-0.9095D-03	-0.3463D-03	0.1310D 00
U(1,5)	0.1260D-02	-0.1716D-03	-0.3019D-03	0.2693D-03	0.5691D-01
U(3,1)	0.4879D-03	-0.1354D-02	0.3517D-04	-0.5267D-04	0.5360D-02
U(3,3)	0.1100D-02	-0.1979D-03	-0.6252D-03	-0.2018D-03	-0.5920D-02
U(3,5)	-0.2011D-02	-0.1071D-03	-0.2178D-03	0.1854D-03	-0.7914D-04
U(5,1)	0.1004D-04	0.2775D-02	-0.6970D-06	0.1312D-03	0.9787D-03
U(5,3)	0.1441D-04	0.4518D-03	0.1129D-02	0.4449D-03	-0.1653D-02
U(5,5)	-0.4277D-04	0.2638D-03	0.4732D-03	-0.5111D-03	-0.4927D-03
V(1,1)	0.2577D-03	0.7894D-03	-0.4578D-03	-0.7594D-04	0.9553D 00
V(1,3)	0.5885D-03	0.1441D-04	0.2029D-03	0.7408D-04	-0.2114D-01
V(1,5)	-0.1027D-02	0.1248D-04	0.2054D-04	-0.1239D-03	-0.5611D-02
V(3,1)	-0.5321D-02	-0.1783D-05	-0.7351D-04	0.1176D-03	0.4885D-01
V(3,3)	-0.5686D-02	-0.3200D-04	-0.4394D-03	0.2960D-03	0.1964D-01
V(3,5)	0.1194D-01	0.8270D-05	-0.2094D-04	-0.8784D-03	0.5269D-02
V(5,1)	-0.6490D-03	0.3510D-02	-0.2425D-02	-0.2212D-02	0.1912D-01
V(5,3)	-0.6090D-03	0.5724D-03	0.7431D-02	-0.3187D-02	0.9318D-02
V(5,5)	0.1235D-02	0.1187D-03	0.4905D-03	0.8908D-02	0.3768D-02
W(1,1)	0.2349D-02	-0.9921D-02	0.1825D-02	0.5272D-03	-0.1890D-01
W(1,3)	0.1004D-01	-0.7122D-03	-0.1187D-01	0.1997D-02	0.1696D-01
W(1,5)	-0.7510D-01	0.1640D-03	-0.1760D-02	-0.1481D-01	0.1324D-01
W(3,1)	-0.2009D-01	-0.9643D-01	0.1257D-01	0.2556D-02	-0.3090D-02
W(3,3)	-0.1356D 00	-0.1193D-01	-0.7333D-01	0.1449D-01	0.8584D-03
W(3,5)	0.9815D 00	-0.3160D-02	-0.1359D-01	-0.1042D 00	0.9048D-03
W(5,1)	-0.2749D-02	0.9866D 00	-0.1214D 00	-0.2319D-01	-0.2056D-02
W(5,3)	-0.1417D-01	0.1172D 00	0.9776D 00	-0.1373D 00	0.6710D-03
W(5,5)	0.1030D 00	0.3972D-01	0.1336D 00	0.9838D 00	0.4157D-03
PSI(1,1)	-0.1639D-03	0.1344D-01	-0.1454D-02	-0.2086D-03	0.3911D-03
PSI(1,3)	-0.1480D-02	0.1598D-02	0.1179D-01	-0.1331D-02	-0.4014D-03
PSI(1,5)	0.1074D-01	0.5430D-03	0.1619D-02	0.9563D-02	-0.2492D-03
PSI(3,1)	0.4574D-03	0.1325D-01	-0.1557D-02	-0.2753D-03	0.8524D-04
PSI(3,3)	0.4426D-02	0.1533D-02	0.1222D-01	-0.1498D-02	-0.1227D-04
PSI(3,5)	-0.3170D-01	0.4854D-03	0.1692D-02	0.1083D-01	-0.1750D-04
PSI(5,1)	0.2825D-04	-0.3625D-01	0.4271D-02	0.7253D-03	0.6231D-04
PSI(5,3)	0.2093D-03	-0.4332D-02	-0.3470D-01	0.4575D-02	-0.1751D-04
PSI(5,5)	-0.1450D-02	-0.1485D-02	-0.4736D-02	-0.3271D-01	-0.8464D-05
PHI(1,1)	0.2239D-03	-0.8564D-03	0.4702D-03	0.2923D-03	0.2008D-02
PHI(1,3)	0.3580D-04	-0.1296D-03	0.4690D-04	0.4814D-04	0.2919D-03
PHI(1,5)	0.9367D-05	-0.4393D-04	0.2147D-04	0.1071D-04	0.1000D-03
PHI(3,1)	0.2787D-02	-0.1132D-02	0.6560D-03	0.4351D-03	0.1928D-03
PHI(3,3)	0.4789D-03	-0.1717D-03	0.5548D-04	0.7346D-04	0.2801D-04
PHI(3,5)	0.8203D-04	-0.5806D-04	0.2885D-04	0.1412D-04	0.9563D-05
PHI(5,1)	0.6433D-03	-0.8692D-03	0.6981D-03	0.7784D-03	0.8307D-04
PHI(5,3)	0.1134D-03	-0.1328D-03	0.3481D-04	0.1415D-03	0.1183D-04
PHI(5,5)	0.1626D-04	-0.4474D-04	0.2817D-04	0.1572D-04	0.4067D-05

Figure II-2. Continued

FREQUENCY =	0.1120D 01	0.1187D 01	0.1269D 01	0.1511D 01	0.1579D 01
GEN COORD	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE
U(1,1)	0.8121D 00	0.2292D 00	-0.4630D 00	-0.7549D-01	0.2529D-02
U(1,3)	0.1366D-01	0.8183D 00	0.2911D 00	0.1139D 00	0.4436D-01
U(1,5)	0.7126D-02	0.5416D-01	-0.1278D 00	0.9060D 00	-0.8003D-01
U(3,1)	-0.6780D-01	0.5293D-01	-0.1187D 00	-0.3495D-01	0.4545D-01
U(3,3)	0.3938D-01	-0.7197D-01	0.1938D-01	0.4800D-01	-0.3766D 00
U(3,5)	0.3639D-01	-0.8975D-02	0.7164D-01	0.1731D-01	0.8397D-01
U(5,1)	-0.4128D-02	0.2121D-02	0.5057D-03	0.6263D-02	-0.7437D-03
U(5,3)	0.1984D-02	-0.1136D-01	-0.6566D-02	0.2400D-02	0.1182D-01
U(5,5)	0.2008D-02	0.2776D-02	0.1770D-02	-0.2125D-01	-0.5247D-02
V(1,1)	0.1807D 00	-0.5085D-01	-0.1998D 00	-0.9100D-01	-0.3401D-02
V(1,3)	0.2712D-01	-0.3572D 00	-0.2225D 00	0.2235D 00	0.5447D 00
V(1,5)	0.4143D-02	0.3850D-02	0.2838D-01	-0.2259D 00	0.3027D-01
V(3,1)	0.5348D 00	-0.3239D 00	0.7312D 00	0.1250D 00	-0.4455D-01
V(3,3)	0.3081D-01	0.1587D 00	0.1837D 00	-0.2193D-01	0.7322D 00
V(3,5)	-0.3804D-02	-0.4615D-02	-0.2747D-01	0.1377D 00	-0.5957D-01
V(5,1)	0.1101D 00	-0.4055D-01	0.5683D-01	-0.5520D-01	-0.7080D-02
V(5,3)	0.1084D-01	0.6866D-01	0.4629D-01	-0.1169D-01	0.2921D-01
V(5,5)	0.3864D-02	-0.1291D-02	-0.4763D-02	0.9280D-01	0.2304D-02
W(1,1)	-0.6669D-02	-0.1579D-02	0.3186D-02	0.1025D-02	0.9927D-03
W(1,3)	0.8223D-03	0.6656D-02	0.3458D-02	-0.5754D-02	-0.9949D-02
W(1,5)	0.1412D-02	-0.3626D-02	-0.3370D-02	0.5109D-02	0.1800D-02
W(3,1)	-0.1969D-02	0.4239D-02	-0.6755D-02	-0.6426D-03	0.1145D-02
W(3,3)	0.3642D-02	-0.3726D-02	0.1186D-02	C.1371D-02	-0.1031D-01
W(3,5)	0.4174D-02	-0.4152D-03	0.5659D-02	-0.1959D-02	0.3960D-02
W(5,1)	0.1375D-02	0.1537D-02	-0.2080D-02	0.1396D-03	0.1944D-03
W(5,3)	0.9000D-03	-0.6921D-05	0.1066D-03	0.4917D-03	-0.1365D-02
W(5,5)	0.8692D-03	0.4198D-03	0.7272D-03	-0.1229D-02	0.1249D-03
PSI(1,1)	0.2254D-03	0.1426D-03	-0.2292D-03	0.2404D-05	0.1499D-03
PSI(1,3)	0.4686D-04	-0.2514D-03	-0.1181D-03	0.2483D-03	0.2083D-03
PSI(1,5)	0.2447D-04	0.9149D-04	0.1279D-03	-0.2116D-03	-0.6171D-04
PSI(3,1)	0.4807D-04	-0.1399D-03	0.2087D-03	0.1864D-04	0.6768D-05
PSI(3,3)	-0.1268D-03	0.1459D-03	-0.2221D-04	-0.4313D-04	0.4994D-03
PSI(3,5)	-0.1284D-03	0.5457D-05	-0.1783D-03	0.7180D-04	-0.1919D-03
PSI(5,1)	-0.6643D-04	-0.4745D-04	0.6287D-04	-0.1929D-05	0.3641D-05
PSI(5,3)	-0.2328D-04	-0.1030D-04	-0.1727D-06	-0.1598D-04	0.3531D-04
PSI(5,5)	-0.1965D-04	-0.1765D-04	-0.1314D-04	0.3836D-04	-0.2286D-05
PHI(1,1)	0.6085D-03	-0.1137D-04	-0.4186D-03	-0.6990D-03	-0.2142D-02
PHI(1,3)	0.9104D-04	-0.9064D-05	-0.6733D-04	-0.1005D-03	-0.3141D-03
PHI(1,5)	0.3081D-04	-0.6873D-06	-0.2130D-04	-0.3641D-04	-0.1093D-03
PHI(3,1)	0.7886D-03	-0.5299D-03	0.1166D-02	0.1972D-03	-0.6553D-03
PHI(3,3)	0.1174D-03	-0.7681D-04	0.1766D-03	0.2937D-04	-0.8770D-04
PHI(3,5)	0.3980D-04	-0.2676D-04	0.5928D-04	0.9989D-05	-0.3314D-04
PHI(5,1)	0.1559D-03	-0.7968D-04	0.1248D-03	-0.6820D-04	-0.1364D-03
PHI(5,3)	0.2311D-04	-0.1105D-04	0.1911D-04	-0.1049D-04	-0.1898D-04
PHI(5,5)	0.7833D-05	-0.4015D-05	0.6518D-05	-0.3584D-05	-0.6953D-05

Figure II-2. Continued

FREQUENCY =	0.1642D 01	0.1676D 01	0.1732D 01	0.1875D 01	0.1953D 01
GEN COORD	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE
U(1,1)	0.2023D-04	-0.5265D-01	-0.4058D-01	0.8669D-04	-0.3734D-02
U(1,3)	-0.1944D-02	0.4182D 00	-0.1013D 00	-0.3378D-04	0.9937D-02
U(1,5)	0.1358D-02	-0.2205D 00	0.9846D-01	0.2800D-04	0.7827D-01
U(3,1)	0.2572D-03	-0.2834D-01	0.9566D-01	0.6072D-03	0.1202D-01
U(3,3)	-0.2417D-02	0.3310D 00	-0.6512D-01	-0.5477D-03	0.5984D-01
U(3,5)	0.5529D-03	-0.7599D-01	-0.4553D-01	0.2262D-03	0.7264D 00
U(5,1)	0.3677D-04	-0.1049D-01	-0.8831D-01	-0.5919D-05	-0.6672D-02
U(5,3)	0.2144D-03	-0.4246D-01	-0.2754D-01	0.1054D-03	0.1137D-01
U(5,5)	-0.1391D-03	0.2799D-01	0.5195D-01	-0.3341D-04	-0.6826D-01
V(1,1)	-0.1578D-02	-0.3814D-01	-0.2471D-01	0.1382D-03	0.8442D-03
V(1,3)	-0.2018D-02	0.6694D 00	-0.1257D 00	0.1260D-04	0.9673D-02
V(1,5)	-0.7142D-03	0.7671D-01	-0.4066D-01	0.1358D-04	0.2543D-01
V(3,1)	-0.3481D-03	0.1607D-01	-0.9716D-01	-0.1390D-02	-0.6175D-01
V(3,3)	0.3420D-02	-0.3741D 00	0.7099D-01	0.1039D-03	-0.1103D 00
V(3,5)	-0.1114D-03	0.3570D-02	0.4679D-01	-0.4210D-03	-0.6459D 00
V(5,1)	-0.5497D-03	0.1436D 00	0.8895D 00	-0.1157D-03	0.6690D-01
V(5,3)	-0.7602D-03	0.1933D 00	0.3497D 00	-0.3484D-03	-0.2610D-02
V(5,5)	0.2025D-03	-0.3239D-01	0.5783D-01	-0.6316D-05	0.1378D 00
W(1,1)	0.1046D-01	0.1451D-02	0.4177D-04	0.2772D-02	0.3068D-04
W(1,3)	-0.4533D-02	-0.1130D-01	0.1832D-02	-0.5434D-03	-0.4830D-04
W(1,5)	-0.6322D-02	0.9726D-03	0.2026D-03	-0.2776D-03	-0.2433D-03
W(3,1)	0.2982D-02	-0.5653D-03	-0.1389D-03	0.9751D-04	-0.2980D-03
W(3,3)	-0.1070D-02	0.4426D-02	-0.1126D-02	-0.5760D-03	-0.3400D-03
W(3,5)	-0.1118D-02	-0.1379D-02	-0.6605D-03	-0.1578D-02	0.8105D-02
W(5,1)	0.2883D-02	-0.5239D-03	-0.3085D-02	0.1080D-02	-0.1354D-03
W(5,3)	-0.9224D-03	-0.8308D-03	-0.1345D-02	-0.6633D-03	0.4854D-03
W(5,5)	-0.9066D-03	0.1322D-02	0.2309D-02	-0.9181D-03	-0.8900D-03
PSI(1,1)	-0.9711D-01	-0.5759D-03	-0.4180D-04	0.9322D-01	-0.6432D-04
PSI(1,3)	0.3014D-01	0.8037D-03	-0.2113D-03	-0.2036D-01	0.4415D-04
PSI(1,5)	0.2784D-01	0.1337D-03	-0.7052D-05	-0.1497D-01	0.7624D-04
PSI(3,1)	-0.1641D-01	-0.9826D-04	0.6574D-06	-0.8994D-01	0.5887D-04
PSI(3,3)	0.5454D-02	-0.1605D-03	0.1291D-04	0.3489D-01	0.4080D-05
PSI(3,5)	0.5465D-02	0.1145D-03	0.4084D-04	0.3936D-01	-0.4723D-03
PSI(5,1)	-0.3993D-02	-0.1119D-04	0.5757D-04	-0.3858D-02	-0.1929D-05
PSI(5,3)	0.1382D-02	0.6773D-04	0.8122D-04	0.2128D-02	-0.1836D-04
PSI(5,5)	0.1492D-02	-0.4096D-04	-0.5655D-04	0.3051D-02	0.5228D-04
PHI(1,1)	0.9751D 00	0.4587D-02	-0.3221D-03	-0.1021D 00	0.1953D-04
PHI(1,3)	0.1487D 00	0.7129D-03	-0.5121D-04	-0.1561D-01	0.2789D-05
PHI(1,5)	0.5011D-01	0.2374D-03	-0.1692D-04	-0.5248D-02	0.1234D-05
PHI(3,1)	0.1069D 00	0.8896D-03	-0.1562D-03	0.9630D 00	-0.4499D-03
PHI(3,3)	0.1633D-01	0.1313D-03	-0.2269D-04	0.1477D 00	-0.6978D-04
PHI(3,5)	0.5503D-02	0.4548D-04	-0.7538D-05	0.4961D-01	-0.2419D-04
PHI(5,1)	0.3464D-01	0.3481D-03	0.1106D-02	0.1303D 00	0.5540D-04
PHI(5,3)	0.5323D-02	0.5599D-04	0.1716D-03	0.2002D-01	0.8748D-05
PHI(5,5)	0.1792D-02	0.1833D-04	0.5702D-04	0.6725D-02	0.3012D-05

Figure II-2. Continued

FREQUENCY =	0.2071D 01	0.2100D 01	0.2166D 01	0.2169D 01	0.2291D 01
GEN CCORD	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE
U(1,1)	-0.1063D-03	0.2889D-01	-0.5649D-02	0.4465D-04	0.6909D-04
U(1,3)	0.6777D-04	-0.1464D 00	-0.1536D-01	-0.3088D-03	-0.1999D-03
U(1,5)	-0.1910D-04	0.2392D-01	0.9553D-03	0.6192D-04	-0.5526D-04
U(3,1)	-0.2247D-03	-0.1355D 00	0.9618D 00	0.4938D-03	0.1705D-03
U(3,3)	-0.3648D-04	0.1306D 00	0.1844D 00	0.1523D-03	0.6255D-03
U(3,5)	-0.4697D-05	0.3811D-01	0.3213D-01	0.5339D-04	0.9089D-04
U(5,1)	-0.1023D-03	0.7573D-01	0.2018D-01	0.9535D-04	0.2679D-03
U(5,3)	0.1220D-03	-0.3296D 00	-0.3392D-01	-0.4968D-03	-0.5690D-03
U(5,5)	-0.1313D-04	0.3622D-01	0.3193D-02	0.4608D-04	0.3418D-03
V(1,1)	-0.1914D-03	0.2299D-01	-0.1071D-01	0.1427D-03	0.7646D-04
V(1,3)	0.5859D-04	-0.8308D-01	-0.1096D-01	-0.5316D-03	-0.1516D-03
V(1,5)	-0.3071D-04	-0.2501D-01	-0.5629D-02	0.4107D-04	-0.6645D-03
V(3,1)	0.6576D-04	-0.2386D-01	0.1377D 00	0.2833D-04	0.1044D-03
V(3,3)	-0.4086D-04	0.1121D 00	0.4912D-01	0.1491D-03	0.3697D-03
V(3,5)	-0.3965D-05	-0.1586D-01	0.3425D-01	0.2325D-04	-0.1171D-03
V(5,1)	0.9673D-04	-0.3448D 00	-0.1059D 00	-0.3112D-03	-0.1596D-02
V(5,3)	-0.3469D-03	0.8272D 00	0.6171D-01	0.1046D-02	0.8855D-03
V(5,5)	-0.1814D-04	-0.1148D-02	-0.1157D-01	0.8578D-05	-0.6988D-03
W(1,1)	0.2469D-01	-0.2435D-03	0.4333D-03	-0.2988D-02	-0.2315D-02
W(1,3)	0.2899D-02	0.1040D-02	0.1432D-03	0.2145D-01	0.2192D-02
W(1,5)	0.7023D-03	0.2383D-03	0.4999D-04	0.3181D-02	-0.2384D-02
W(3,1)	-0.3363D-02	0.8634D-03	-0.4380D-02	-0.2149D-03	0.2858D-02
W(3,3)	-0.6229D-03	-0.2179D-02	-0.8553D-03	-0.1203D-02	-0.8282D-03
W(3,5)	-0.2433D-03	0.3699D-03	-0.2591D-03	-0.8292D-04	-0.1032D-02
W(5,1)	-0.1450D-01	0.1367D-02	0.1256D-02	0.1777D-02	-0.1231D-02
W(5,3)	-0.1811D-02	-0.6901D-02	-0.4642D-03	-0.1240D-01	-0.4480D-03
W(5,5)	-0.5961D-03	0.1125D-02	0.1175D-03	-0.1771D-02	0.1859D-02
PSI(1,1)	0.9492D 00	0.7474D-03	0.2440D-03	-0.1162D 00	-0.1017D 00
PSI(1,3)	0.1284D 00	-0.1419D-02	-0.6508D-03	0.9407D 00	0.1048D 00
PSI(1,5)	0.4508D-01	-0.4013D-04	-0.4771D-04	0.1317D 00	-0.1241D 00
PSI(3,1)	0.2359D 00	0.2481D-04	0.5276D-03	-0.3848D-01	0.1721D-01
PSI(3,3)	0.2594D-01	-0.2299D-03	-0.1062D-03	0.2548D 00	0.1659D-01
PSI(3,5)	0.6091D-02	0.1685D-04	0.5851D-05	0.3789D-01	-0.5394D-01
PSI(5,1)	0.4316D-01	0.9227D-04	-0.1018D-04	0.1142D-02	-0.9326D-01
PSI(5,3)	0.9383D-02	0.2141D-03	-0.3339D-04	0.5504D-01	0.3988D-01
PSI(5,5)	0.5505D-02	-0.1018D-03	-0.2932D-04	0.4449D-02	0.2910D-01
PHI(1,1)	0.9503D-01	0.9233D-04	0.8507D-04	-0.4460D-01	-0.3025D-01
PHI(1,3)	0.1455D-01	0.1585D-04	0.1450D-04	-0.8877D-02	-0.4978D-02
PHI(1,5)	0.4892D-02	0.5258D-05	0.5040D-05	-0.2521D-02	-0.1527D-02
PHI(3,1)	-0.6720D-01	0.4793D-03	-0.1886D-03	0.2670D-01	-0.1197D 00
PHI(3,3)	-0.1034D-01	0.7598D-04	-0.2930D-04	0.4247D-02	-0.1855D-01
PHI(3,5)	-0.3465D-02	0.2481D-04	-0.9449D-05	0.1392D-02	-0.6178D-02
PHI(5,1)	0.8456D-01	-0.1420D-02	-0.5123D-03	-0.9826D-01	0.9531D 00
PHI(5,3)	0.1313D-01	-0.2095D-03	-0.7907D-04	-0.1412D-01	0.1484D 00
PHI(5,5)	0.4384D-02	-0.7270D-04	-0.2620D-04	-0.4968D-02	0.4942D-01

Figure II-2. Continued

FREQUENCY =	0.2354D 01	0.2362D 01	0.2412D 01	0.2460D 01	0.2891D 01
GEN CCORD	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE
U(1,1)	-0.8210D-04	-0.2418D-02	0.7895D-02	0.1089D-01	0.3934D-04
U(1,3)	-0.5439D-03	-0.1886D-01	0.1608D-01	-0.7136D-01	0.2204D-05
U(1,5)	0.3870D-02	0.1213D 00	-0.2216D 00	0.3720D-02	0.1157D-04
U(3,1)	-0.6861D-04	-0.3489D-02	-0.6896D-02	-0.1452D 00	-0.2335D-03
U(3,3)	0.4062D-03	0.2088D-01	0.6947D-01	0.8144D 00	-0.1617D-04
U(3,5)	0.7747D-03	0.1635D-01	-0.8469D-01	0.8305D-01	-0.5795D-04
U(5,1)	0.2306D-03	0.1081D-01	0.1640D-01	-0.2893D-01	-0.1153D-03
U(5,3)	0.1076D-03	0.1040D-01	0.3467D-01	0.1175D 00	-0.7819D-05
U(5,5)	-0.4716D-02	-0.2618D 00	-0.4510D 00	0.4353D-01	-0.1105D-04
V(1,1)	-0.6193D-04	-0.3583D-02	0.7532D-02	0.1474D-01	0.2209D-04
V(1,3)	-0.3049D-03	-0.1376D-01	0.8086D-02	-0.5509D-01	0.1424D-06
V(1,5)	0.2164D-01	0.8073D 00	-0.5226D 00	0.5878D-02	0.9570D-05
V(3,1)	-0.6964D-04	-0.4324D-02	-0.9714D-02	-0.1395D 00	-0.1359D-03
V(3,3)	0.2473D-03	0.1116D-01	0.3332D-01	0.4683D 00	0.2434D-05
V(3,5)	0.4477D-02	0.1922D 00	0.4949D-01	0.8913D-01	-0.5842D-04
V(5,1)	-0.4264D-03	-0.1300D-01	-0.1831D-01	0.7380D-01	0.1428D-04
V(5,3)	0.5628D-03	0.2367D-01	0.7124D-02	-0.1626D 00	-0.3345D-05
V(5,5)	0.8904D-02	0.4743D 00	0.6754D 00	-0.7119D-01	0.2175D-05
W(1,1)	-0.9240D-03	0.3256D-03	-0.2136D-03	-0.1361D-03	-0.7015D-02
W(1,3)	-0.2175D-02	0.1148D-02	-0.6395D-03	0.7890D-03	-0.1276D-02
W(1,5)	0.1732D-01	-0.1106D-01	0.6715D-02	-0.1191D-03	-0.4490D-03
W(3,1)	0.3269D-03	0.1025D-03	0.1081D-03	0.1180D-02	0.4005D-01
W(3,3)	-0.4214D-03	0.5577D-04	-0.4171D-03	-0.6203D-02	0.7195D-02
W(3,5)	0.2058D-02	-0.2598D-02	-0.7042D-03	-0.6822D-03	0.2449D-02
W(5,1)	0.1575D-04	0.1641D-03	0.2068D-03	-0.2858D-03	-0.3431D-02
W(5,3)	0.1348D-02	0.2650D-03	0.5684D-03	0.1241D-02	-0.7031D-03
W(5,5)	-0.9357D-02	-0.4312D-02	-0.6226D-02	0.5165D-03	-0.2804D-03
PSI(1,1)	-0.3753D-01	0.8537D-03	-0.1526D-03	0.6472D-04	-0.2257D 00
PSI(1,3)	-0.1218D 00	0.2977D-02	-0.3957D-03	-0.1694D-03	-0.4340D-01
PSI(1,5)	0.9274D 00	-0.2256D-01	0.3295D-02	-0.1208D-03	-0.1694D-01
PSI(3,1)	-0.5243D-02	0.1430D-03	-0.3676D-04	-0.2182D-03	0.9464D 00
PSI(3,3)	-0.4267D-01	0.1053D-02	-0.9796D-04	0.7814D-03	0.1759D 00
PSI(3,5)	0.3051D 00	-0.7431D-02	0.1214D-02	0.7553D-04	0.6357D-01
PSI(5,1)	-0.1586D-01	0.2911D-03	-0.9384D-04	0.5412D-04	0.6183D-01
PSI(5,3)	-0.5146D-02	0.1476D-03	-0.3632D-04	-0.8406D-04	0.9441D-02
PSI(5,5)	0.8167D-01	-0.1749D-02	0.6498D-03	-0.5334D-04	0.2298D-02
PHI(1,1)	-0.3030D-01	0.8719D-03	-0.2081D-03	-0.4448D-05	-0.1585D-01
PHI(1,3)	-0.4263D-02	0.1227D-03	-0.2972D-04	-0.2637D-06	-0.2394D-02
PHI(1,5)	-0.2027D-02	0.5917D-04	-0.1422D-04	-0.3396D-06	-0.8092D-03
PHI(3,1)	-0.1815D-01	0.3992D-03	-0.5412D-04	0.2334D-03	0.9880D-01
PHI(3,3)	-0.2758D-02	0.6055D-04	-0.8354D-05	0.4071D-04	0.1501D-01
PHI(3,5)	-0.9958D-03	0.2282D-04	-0.2556D-05	0.1256D-04	0.5062D-02
PHI(5,1)	0.1375D 00	-0.2447D-02	0.7157D-03	-0.3660D-03	-0.1986D-01
PHI(5,3)	0.2120D-01	-0.3759D-03	0.1106D-03	-0.5837D-04	-0.3097D-02
PHI(5,5)	0.7376D-02	-0.1318D-03	0.3895D-04	-0.1979D-04	-0.1030D-02

Figure II-2. Continued

FREQUENCY =	0.2953D 01	0.2979D 01	0.3165D 01	0.3246D 01	0.3445D 01
GEN COORD	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE
U(1,1)	0.3965D-02	-0.1279D-04	-0.8179D-05	-0.1660D-02	0.3006D-02
U(1,3)	0.2184D-01	0.7884D-04	-0.3918D-04	-0.3854D-03	-0.1545D-01
U(1,5)	-0.1425D 00	-0.3476D-04	0.2567D-03	-0.2099D-03	-0.2824D-02
U(3,1)	-0.1878D-01	0.6715D-04	0.3362D-04	-0.3828D-02	0.7533D-02
U(3,3)	-0.1010D 00	-0.3935D-03	0.1572D-03	-0.3545D-03	-0.3731D-01
U(3,5)	0.6539D 00	0.1519D-03	-0.1021D-02	0.2294D-03	-0.6247D-02
U(5,1)	-0.4566D-02	0.3000D-04	0.1104D-04	0.9758D 00	-0.1787D 00
U(5,3)	-0.1989D-01	-0.1514D-03	0.3982D-04	0.1877D 00	0.9021D 00
U(5,5)	0.1236D 00	0.1596D-04	-0.2648D-03	0.6293D-01	0.1584D 00
V(1,1)	0.5569D-02	-0.2455D-04	-0.1799D-04	-0.2528D-02	0.3661D-02
V(1,3)	0.1445D-01	0.7319D-04	-0.3543D-04	-0.5845D-03	-0.9182D-02
V(1,5)	-0.1121D 00	-0.2388D-04	0.2509D-03	-0.3619D-03	-0.1785D-02
V(3,1)	-0.3614D-01	0.1461D-03	0.1009D-03	-0.9329D-02	0.1616D-01
V(3,3)	-0.9205D-01	-0.4233D-03	0.1978D-03	-0.1677D-02	-0.3895D-01
V(3,5)	0.7090D 00	0.1537D-03	-0.1383D-02	-0.6333D-03	-0.7007D-02
V(5,1)	0.1370D-02	-0.7754D-05	0.8220D-06	0.8989D-01	-0.1314D 00
V(5,3)	0.1177D-02	-0.1607D-05	0.1042D-04	0.1956D-01	0.3234D 00
V(5,5)	-0.6412D-02	-0.2208D-05	-0.7440D-04	0.1067D-01	0.6071D-01
W(1,1)	-0.3740D-04	0.1323D-02	0.2887D-03	0.8851D-04	-0.2265D-04
W(1,3)	-0.1881D-03	-0.6902D-02	0.8814D-03	0.1575D-04	0.1090D-03
W(1,5)	0.1204D-02	-0.8482D-03	-0.6839D-02	0.5204D-05	0.1820D-04
W(3,1)	0.2322D-03	-0.7211D-02	-0.1447D-02	0.2939D-03	-0.9199D-04
W(3,3)	0.1167D-02	0.3724D-01	-0.4295D-02	0.4908D-04	0.4360D-03
W(3,5)	-0.7451D-02	0.4632D-02	0.3349D-01	0.1132D-04	0.6837D-04
W(5,1)	0.4702D-05	0.4225D-03	-0.5751D-05	-0.3072D-02	0.7813D-03
W(5,3)	0.1976D-04	-0.2655D-02	0.1807D-03	-0.5380D-03	-0.3735D-02
W(5,5)	-0.1064D-03	-0.2947D-03	-0.1204D-02	-0.1610D-03	-0.6064D-03
PSI(1,1)	-0.6375D-05	0.4649D-01	0.1140D-01	-0.4580D-05	0.3626D-05
PSI(1,3)	0.1553D-03	-0.2557D 00	0.4171D-01	-0.1288D-05	-0.1562D-04
PSI(1,5)	-0.6272D-03	-0.2992D-01	-0.3154D 00	-0.8815D-06	-0.2651D-05
PSI(3,1)	0.3914D-04	-0.1753D 00	-0.3628D-01	0.6064D-04	-0.7362D-05
PSI(3,3)	-0.5217D-03	0.9368D 00	-0.1221D 00	0.1217D-04	0.3184D-04
PSI(3,5)	0.1901D-02	0.1132D 00	0.9345D 00	0.4979D-05	0.5071D-05
PSI(5,1)	0.5147D-05	-0.1414D-01	-0.4349D-02	0.4285D-03	-0.1587D-03
PSI(5,3)	-0.2777D-04	0.6442D-01	-0.9450D-02	0.7164D-04	0.6523D-03
PSI(5,5)	0.8932D-04	0.8706D-02	0.7714D-01	0.2091D-04	0.1073D-03
PHI(1,1)	0.2077D-05	0.1077D-01	0.7860D-02	-0.2804D-06	-0.6965D-06
PHI(1,3)	0.1618D-06	0.2948D-02	0.9365D-03	-0.6472D-07	0.2846D-07
PHI(1,5)	-0.4795D-06	0.6930D-03	0.7325D-03	0.1024D-07	-0.3191D-07
PHI(3,1)	0.6909D-05	-0.6296D-01	-0.4118D-01	0.5393D-05	-0.7938D-05
PHI(3,3)	0.3381D-05	-0.1666D-01	-0.5041D-02	0.8132D-06	-0.1679D-05
PHI(3,5)	-0.4609D-06	-0.3993D-02	-0.3736D-02	0.3777D-06	-0.5861D-06
PHI(5,1)	0.7491D-05	0.9948D-02	0.3738D-02	-0.8876D-05	0.3995D-04
PHI(5,3)	0.8658D-06	0.2043D-02	0.5452D-03	-0.2143D-05	0.6113D-05
PHI(5,5)	0.3835D-06	0.5692D-03	0.2514D-03	-0.2149D-05	0.2033D-05

Figure II-2. Continued

FREQUENCY =	0.3809D 01	0.3826D 01	0.3885D 01	0.4017D 01	0.1562D 02
GEN COORD	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE
U(1,1)	0.8904D-03	0.1093D-04	-0.3628D-05	-0.1839D-05	-0.6857D-08
U(1,3)	0.5712D-02	0.1750D-05	0.1917D-04	-0.8548D-05	0.1421D-07
U(1,5)	-0.3150D-01	0.3194D-05	-0.6024D-05	0.5139D-04	0.5628D-07
U(3,1)	0.2249D-02	0.2801D-04	-0.9213D-05	-0.3357D-05	0.2566D-07
U(3,3)	0.1438D-01	0.4132D-05	0.4913D-04	-0.2131D-04	-0.5630D-06
U(3,5)	-0.7824D-01	0.5116D-05	-0.1502D-04	0.1295D-03	-0.9135D-07
U(5,1)	-0.2356D-01	-0.2948D-03	0.9047D-04	0.4006D-04	0.9028D-07
U(5,3)	-0.1505D 00	-0.4231D-04	-0.4744D-03	0.1813D-03	-0.1113D-06
U(5,5)	0.8197D 00	-0.7541D-04	0.1431D-03	-0.1100D-02	-0.1653D-06
V(1,1)	0.1182D-02	0.2243D-05	-0.3176D-05	-0.2671D-05	0.5967D-05
V(1,3)	0.3141D-02	0.1973D-06	0.8673D-05	-0.5130D-05	-0.1365D-04
V(1,5)	-0.1519D-01	0.1607D-05	-0.2981D-05	0.2596D-04	0.2946D-06
V(3,1)	0.5902D-02	0.1022D-04	-0.1495D-04	-0.1272D-04	0.7524D-06
V(3,3)	0.1563D-01	0.3128D-06	0.4080D-04	-0.2526D-04	-0.1872D-05
V(3,5)	-0.7462D-01	0.3824D-05	-0.1519D-04	0.1293D-03	-0.3806D-06
V(5,1)	-0.4166D-01	-0.7574D-04	0.1084D-03	0.8826D-04	0.2314D-06
V(5,3)	-0.1103D 00	-0.3542D-05	-0.2923D-03	0.1724D-03	-0.4628D-06
V(5,5)	0.5268D 00	-0.4161D-04	0.1049D-03	-0.8751D-03	0.7965D-07
W(1,1)	-0.4152D-05	-0.1166D-02	0.2496D-03	0.5621D-04	-0.7570D-04
W(1,3)	-0.2534D-04	-0.2468D-03	-0.1135D-02	0.1323D-03	0.4888D-03
W(1,5)	0.1333D-03	-0.8902D-04	-0.1238D-03	-0.1106D-02	-0.1751D-04
W(3,1)	-0.2017D-04	-0.4655D-02	0.1028D-02	0.2477D-03	-0.8340D-05
W(3,3)	-0.1226D-03	-0.9740D-03	-0.4629D-02	0.5671D-03	0.5386D-04
W(3,5)	0.6408D-03	-0.3422D-03	-0.5123D-03	-0.4767D-02	-0.1867D-05
W(5,1)	0.1419D-03	0.3658D-01	-0.7796D-02	-0.1748D-02	-0.3010D-05
W(5,3)	0.8639D-03	0.7689D-02	0.3523D-01	-0.4024D-02	0.1948D-04
W(5,5)	-0.4518D-02	0.2732D-02	0.3878D-02	0.3378D-01	-0.6956D-06
PSI(1,1)	-0.6451D-06	-0.3503D-01	0.8343D-02	0.2236D-02	-0.3208D-03
PSI(1,3)	0.1756D-04	-0.7643D-02	-0.3912D-01	0.5889D-02	0.2071D-02
PSI(1,5)	-0.6748D-04	-0.2907D-02	-0.4098D-02	-0.4816D-01	-0.7617D-04
PSI(3,1)	-0.5854D-05	-0.6817D-01	0.1667D-01	0.4748D-02	-0.2428D-05
PSI(3,3)	0.3012D-04	-0.1443D-01	-0.7621D-01	0.1169D-01	0.1417D-04
PSI(3,5)	-0.8816D-04	-0.5144D-02	-0.8302D-02	-0.9687D-01	0.1456D-05
PSI(5,1)	0.4623D-04	0.9682D 00	-0.2099D 00	-0.4805D-01	-0.2777D-05
PSI(5,3)	-0.4520D-03	0.2070D 00	0.9657D 00	-0.1189D 00	0.1743D-04
PSI(5,5)	0.1556D-02	0.7563D-01	0.1040D 00	0.9842D 00	-0.2360D-06
PHI(1,1)	0.8250D-06	-0.2377D-02	0.1673D-02	0.1263D-02	-0.1542D 00
PHI(1,3)	0.6160D-07	-0.3496D-03	0.6314D-03	0.1303D-03	0.9753D 00
PHI(1,5)	0.1406D-06	-0.1196D-03	0.1253D-03	0.1516D-03	0.1044D 00
PHI(3,1)	0.8335D-06	-0.1016D-01	0.7268D-02	0.5739D-02	-0.1725D-01
PHI(3,3)	0.1389D-06	-0.1513D-02	0.2639D-02	0.6149D-03	0.1092D 00
PHI(3,5)	-0.1936D-06	-0.5153D-03	0.5350D-03	0.6695D-03	0.1169D-01
PHI(5,1)	-0.1690D-04	0.7833D-01	-0.5457D-01	-0.4060D-01	-0.6336D-02
PHI(5,3)	0.1685D-06	0.1162D-01	-0.1999D-01	-0.4344D-02	0.4008D-01
PHI(5,5)	-0.3328D-05	0.3965D-02	-0.4032D-02	-0.4740D-02	0.4299D-02

Figure II-2. Continued

FREQUENCY =	0.1566D 02	0.1572D 02	0.3956D 02	0.3958D 02	0.3960D 02
GEN COORD	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE	MODE SHAPE
U(1,1)	-0.6528D-08	-0.1889D-07	0.2294D-07	0.5089D-07	0.1105D-06
U(1,3)	0.5320D-07	0.5193D-07	-0.1799D-06	0.3788D-07	-0.1303D-06
U(1,5)	-0.1431D-06	0.1123D-06	-0.2064D-07	0.3683D-06	0.2123D-06
U(3,1)	0.2791D-06	-0.5886D-07	-0.4751D-06	-0.3216D-06	-0.3726D-06
U(3,3)	-0.4797D-08	0.8285D-07	-0.2298D-06	-0.7755D-07	0.2968D-06
U(3,5)	0.4882D-07	-0.4723D-06	0.3980D-06	-0.4445D-06	0.1816D-06
U(5,1)	0.1182D-06	0.6118D-06	-0.8333D-07	-0.1114D-06	0.1533D-05
U(5,3)	0.6763D-07	-0.3177D-06	0.1967D-06	0.1626D-07	0.3893D-06
U(5,5)	-0.2647D-06	0.1175D-05	0.1079D-06	0.6376D-06	-0.2081D-07
V(1,1)	-0.6999D-06	-0.1701D-06	0.3118D-06	-0.1900D-08	0.3769D-07
V(1,3)	0.1604D-05	0.3891D-06	0.1726D-06	0.1479D-06	-0.1218D-06
V(1,5)	-0.5682D-06	-0.1057D-06	-0.2258D-05	0.1123D-06	-0.2923D-06
V(3,1)	0.5953D-05	-0.8148D-06	-0.1318D-06	0.2996D-06	-0.2831D-06
V(3,3)	-0.1350D-04	0.2042D-05	0.2241D-06	0.5199D-06	0.2541D-06
V(3,5)	0.4857D-06	0.9263D-07	0.8259D-06	-0.1780D-05	0.3256D-06
V(5,1)	0.8164D-06	0.5974D-05	0.2082D-06	-0.3556D-06	0.4267D-06
V(5,3)	-0.1879D-05	-0.1379D-04	-0.4969D-06	-0.1584D-06	0.3966D-06
V(5,5)	0.7602D-07	0.1131D-05	-0.2667D-07	-0.5516D-07	-0.1893D-05
W(1,1)	0.9093D-05	0.2021D-05	-0.3619D-05	0.1029D-07	-0.1585D-07
W(1,3)	-0.5863D-04	-0.1296D-04	-0.1910D-04	-0.3607D-08	-0.2461D-08
W(1,5)	0.2086D-05	0.4492D-06	0.1258D-03	-0.6921D-07	-0.3737D-07
W(3,1)	-0.7617D-04	0.1158D-04	0.8568D-08	-0.3631D-05	0.2727D-07
W(3,3)	0.4907D-03	-0.7428D-04	-0.2278D-07	-0.1913D-04	-0.3203D-08
W(3,5)	-0.1741D-04	0.2596D-05	-0.3279D-07	0.1261D-03	-0.1323D-06
W(5,1)	-0.1051D-04	-0.7892D-04	-0.3242D-08	0.3988D-08	-0.3756D-05
W(5,3)	0.6769D-04	0.5060D-03	0.8391D-08	0.7423D-08	-0.1918D-04
W(5,5)	-0.2383D-05	-0.1761D-04	-0.4793D-07	-0.1500D-06	0.1266D-03
PSI(1,1)	0.3601D-03	0.1919D-03	-0.1668D-04	0.1387D-04	0.9915D-05
PSI(1,3)	-0.2325D-02	-0.1231D-02	-0.8981D-04	0.7106D-04	0.5551D-04
PSI(1,5)	0.8541D-04	0.4392D-04	0.5928D-03	-0.4700D-03	-0.3668D-03
PSI(3,1)	-0.1007D-02	0.4027D-03	0.4972D-05	-0.4836D-04	0.1203D-04
PSI(3,3)	0.6492D-02	-0.2585D-02	0.2448D-04	-0.2550D-03	0.6244D-04
PSI(3,5)	-0.2354D-03	0.9279D-04	-0.1648D-03	0.1684D-02	-0.4140D-03
PSI(5,1)	-0.8898D-04	-0.1757D-02	0.2324D-05	0.7515D-05	-0.8256D-04
PSI(5,3)	0.5721D-03	0.1128D-01	0.1199D-04	0.3815D-04	-0.4178D-03
PSI(5,5)	-0.1984D-04	-0.4006D-03	-0.7952D-04	-0.2549D-03	0.2759D-02
PHI(1,1)	0.1812D-01	0.3922D-02	-0.3402D-01	0.8016D-05	0.6510D-05
PHI(1,3)	-0.1137D 00	-0.2427D-01	-0.1117D 00	0.3833D-05	0.1892D-05
PHI(1,5)	-0.1217D-01	-0.2595D-02	0.9932D 00	0.3206D-05	-0.1106D-05
PHI(3,1)	-0.1539D 00	0.2259D-01	0.8178D-05	-0.3409D-01	0.2024D-04
PHI(3,3)	0.9659D 00	-0.1397D 00	0.3315D-05	-0.1118D 00	0.8677D-05
PHI(3,5)	0.1034D 00	-0.1496D-01	-0.1245D-05	0.9931D 00	0.6435D-05
PHI(5,1)	-0.2168D-01	-0.1571D 00	0.6735D-05	0.2132D-04	-0.3420D-01
PHI(5,3)	0.1361D 00	0.9713D 00	0.8515D-06	0.4990D-05	-0.1120D 00
PHI(5,5)	0.1460D-01	0.1041D 00	0.2249D-05	-0.2201D-05	0.9931D 00

Figure II-2. Continued

REFERENCES

1. M. E. Raville, En-Shinh Ueng, and Ming-Min Lei, "Natural Frequencies of Vibration of Fixed-Fixed Sandwich Beams," Journal of Applied Mechanics, Trans. ASME, Vol. 83, Series E, 1961, pp 367-371.

2. C. E. S. Ueng, "Natural Frequencies of Vibration of an All-Clamped Rectangular Sandwich Panel," Journal of Applied Mechanics, Trans. ASME, Vol. 88, Series E, 1966, pp 683-684.

3. Alfred M. Freudenthal and Maciej P. Bieniek, "Forced Vibrations of Sandwich Structures," WADD TR 60-307, January 1961.

4. D. J. Mead and A. J. Pretlove, "On the Vibrations of Cylindrically Curved Elastic Sandwich Plates: Part 1: With the Solution for Flat Plates," University of Southampton, A.A.S.U. Report No. 186, August 1961.

5. A. J. Pretlove, "On the Vibrations of Cylindrically Curved Elastic Sandwich Plates: Part II: The Solution for Cylindrical Plates," University of Southampton, A.A.S.U. Report No. 187, February 1962.

6. J. R. Ballentine, H. E. Plumblee, and C. W. Schneider, "Sonic Fatigue in Combined Environment," Air Force Flight Dynamics Laboratory Technical Report AFFDL-TR-66-7, May 1966.

7. Chi-Teh Wang, Applied Elasticity, McGraw-Hill, New York, 1953.

8. R. N. Arnold and G. D. Warburton, "The Flexural Vibrations of Thin Cylinders," Proceedings of the Institution of Mechanical Engineers, Vol. 167, 1953, pp 62-74.

9. R. M. Scruggs, "Transverse Vibration of a Cantilevered Circular Cylindrical Shell," Masters Thesis, Department of Engineering Mechanics, Georgia Institute of Technology, June 1964.

10. J. E. Sweers, "Prediction of Response and Fatigue Life of Honeycomb Sandwich Panels Subjected to Acoustical Excitation," Acoustical Fatigue in Aerospace Structures, Syracuse University Press, Syracuse, New York, 1965.

11. I. S. Sokolnikoff, Mathematical Theory of Elasticity, McGraw-Hill Book Company, New York, 1956.

12. Hu-Nan Chu, "Non-Axisymmetric Vibrations of Honeycomb Sandwich Cylinders," R-60-28, The Martin Company, Denver, November 1960.

13. R. P. Felgar, Jr., "Formulas for Integrals Containing Characteristic Functions of a Vibrating Beam," Circular No. 14, Bureau of Engineering Research, University of Texas, Austin, 1950.

14. R. Courant and D. Hilbert, Methods of Mathematical Physics, Volume I, Interscience Publishers, Inc., New York, 1953.

15. John L. Sewall, "Vibration Analysis of Cylindrically Curved Panels with Simply Supported or Clamped Edges and Comparison with Some Experiments," NASA TN D-3791, Langley Research Center, January 1967.

16. Carl E. Rucker, "Some Experimental Effects of Curvature on Response of Simple Panels to Intense Noise," paper presented at 67th Meeting of the Acoustical Society of America (New York, N.Y.), May 1964.

17. G. B. Warburton, "The Vibration of Rectangular Plates," Proceedings of the Institute of Mechanical Engineers, Vol. 168, 1954.

18. S. Kelsey, R. A. Gellatly, and B. W. Clark, "The Shear Modulus of Foil Honeycomb Cores," Aircraft Engineering, October 1958.